Browney Academy Calculation Policy

What is maths mastery?

Teaching maths for mastery is a transformational approach to maths teaching which stems from high performing Asian nations such as Singapore. When taught to master maths, children develop their mathematical fluency without resorting to rote learning and are able to solve non-routine maths problems without having to memorise procedures.

Concrete, pictorial, abstract (CPA)

Concrete, pictorial, abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths. Developed by American psychologist, Jerome Bruner, the CPA approach is essential to maths teaching in Singapore.
Pupils builds on their existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems.

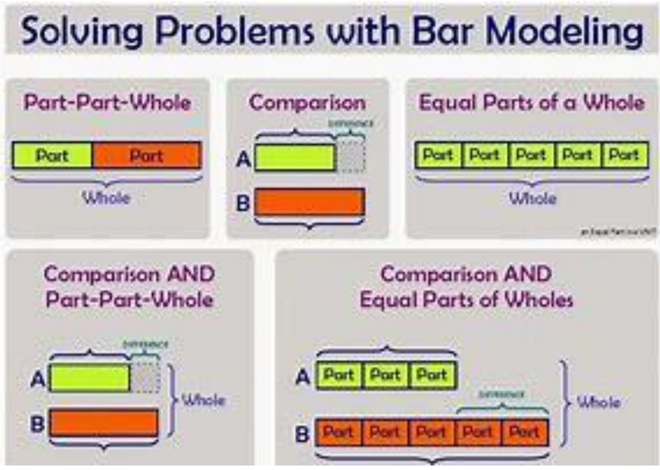
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. This approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem and makes it far easier for them to grasp difficult abstract concepts.

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Pupils will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, –, x, /) to indicate addition, multiplication or division.

Bar modelling

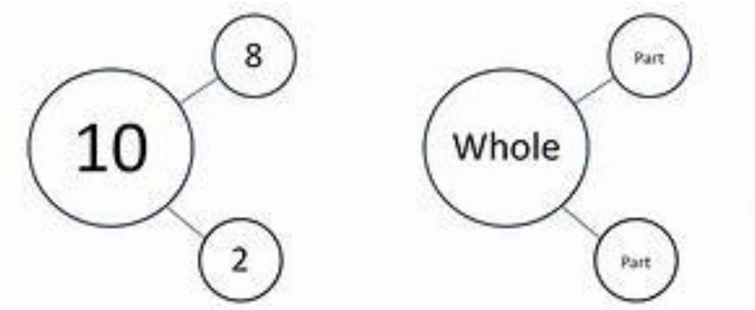
The bar model method is a strategy used by children to visualise mathematical concepts and solve problems. The method is a way to represent a situation in a word problem, usually using rectangles.



Number bonds

Number bonds are a way of showing how numbers can be combined or split up. They are used to reflect the 'part-part-whole' relationship of numbers. A lot of emphasis is put into number bonds from the early year foundation stages so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

The mastery of number bonds is an important foundation required in subsequent mathematical learning and as a basis in the development of mental strategies. A strong number sense allows students to decide what action to take when trying to solve problems in their head.



Year 1
Place Value - Counting

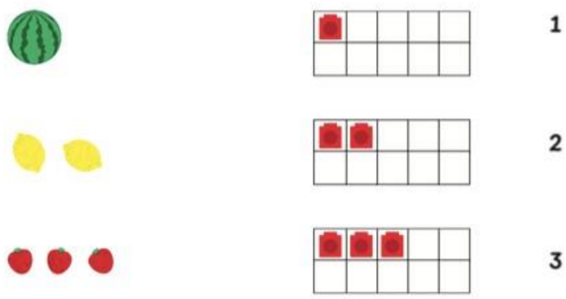
Counting to 10:

We can count on....

We can count back....

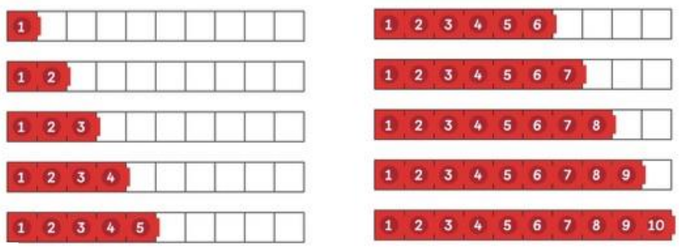
Then we learn about 0.

Counting with objects:



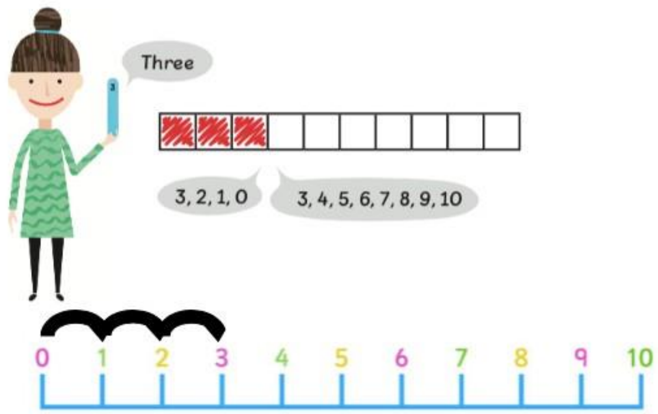
Physical objects | Tens squares

Counting with objects:



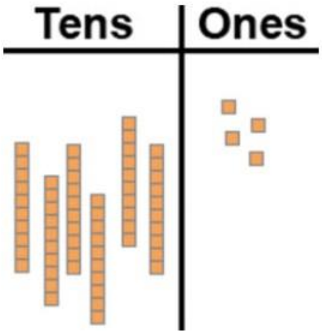
Using multilink cubes

Counting with number lines:



Year 1
Place Value

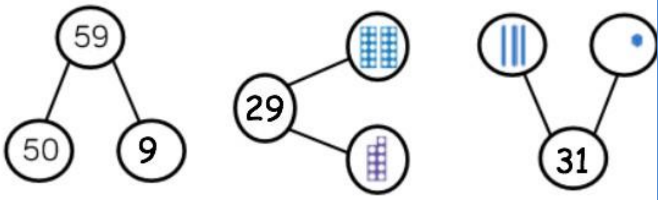
Dienes to represent numbers:



The dienes show **6 tens** and **4 ones**.

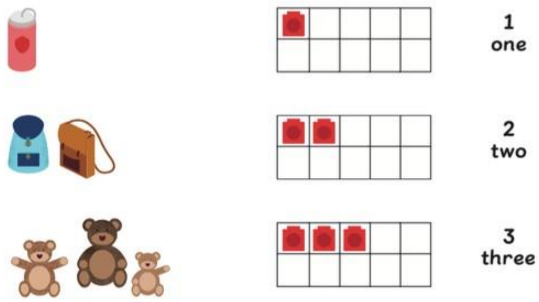
This shows the number **64**.

Number bond method:

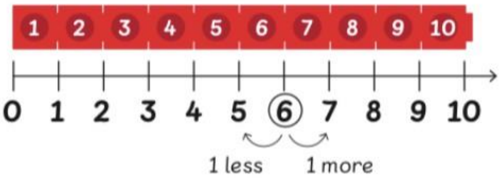


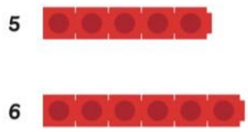
Separating the numbers apart like this is called **partitioning.**

Writing numbers to 10:



Ordering numbers:





We can find **1 more** and **1 less** than.

Comparing numbers:

There are 3 cupcakes.

There are 5 cookies.

There are 7 doughnuts.

Which number is more than the others?
Which number is less than the others?

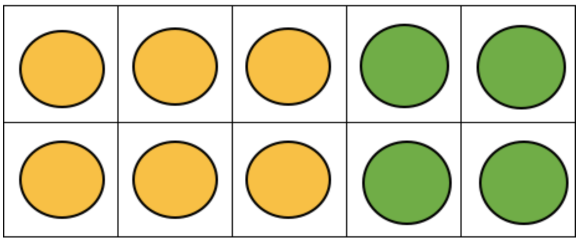
1	2	**3**	4	**5**	6	**7**	8	9	10

7 is more than 5.
7 is more than 3.
7 is the **greatest**.

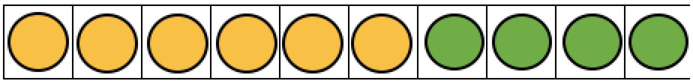
3 is less than 7.
3 is less than 5.
3 is the **smallest**.

Year 1
Addition

Tens frame:



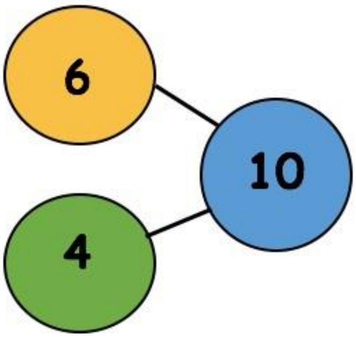
Tens strip:



Count on from the biggest number:

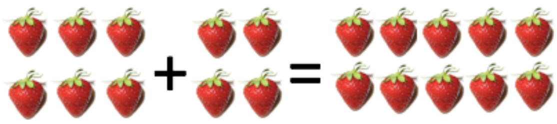
6 + 4 = 10

Number bond method:

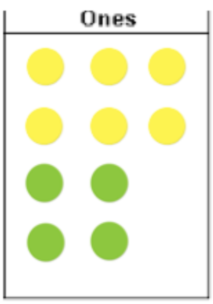


Number bond method:

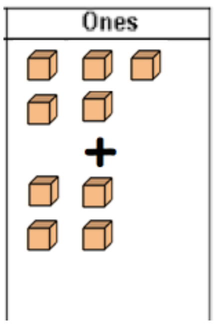
Picture method:



Counters method:



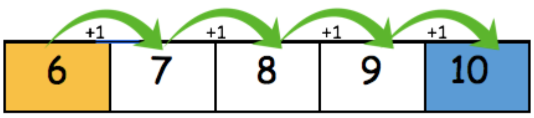
Base 10 method:

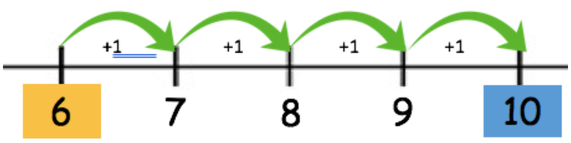


Abstract calculations:

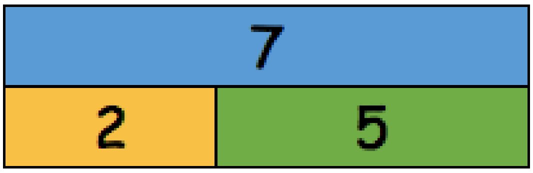
Commutative	Inverse
2 + 5 = 7	7 - 5 = 2
5 + 2 = 7	7 - 2 = 5

Number line method:





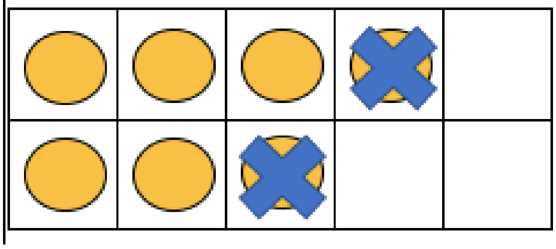
Bar model:



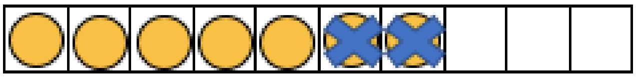
Year 1

Subtraction

Tens frame:



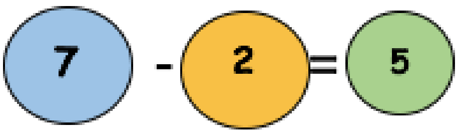
Tens strip:



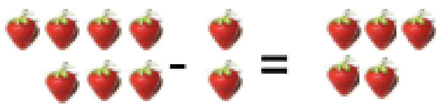
Count back from the biggest number:

7 - 2 = 5

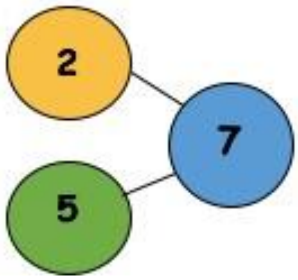
Number bond method:



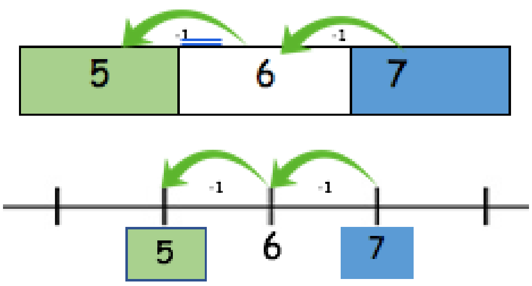
Picture method:



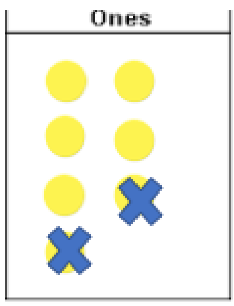
Number bond method:



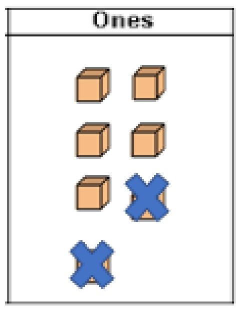
Number line method:



Counters method:

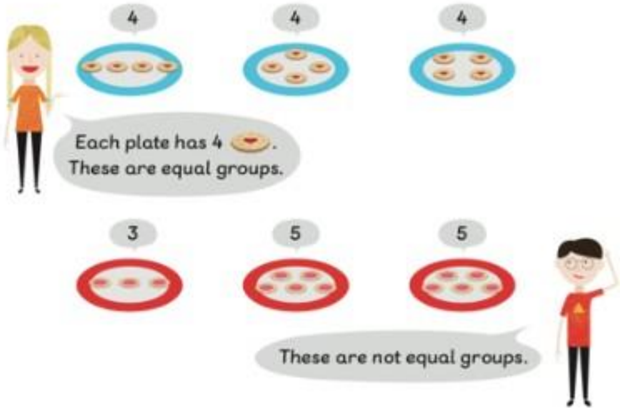


Base 10 method:



Year 1
Multiplication & Division

Making equal groups



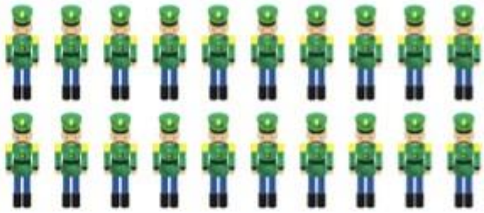
Adding equal groups

There are 4 trays.

4 trays of 5 = 20
4 groups of 5 = 20
4 fives = 20

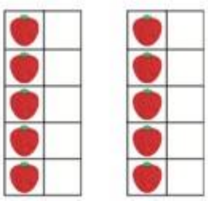
There are 20 altogether.

Making equal rows



There are 10 toy soldiers in one row.
2 tens = 20
There are 20 toy soldiers altogether.

Making doubles



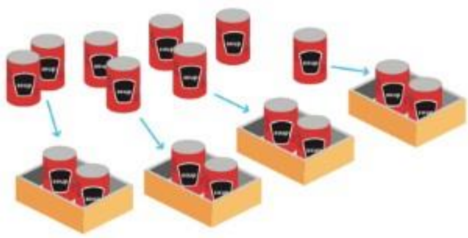
Double 2 = 4

Double 5 = 10

DIVISION

Grouping equally

There are 8 cans.



There are 4 boxes of 2 cans.

Sharing equally

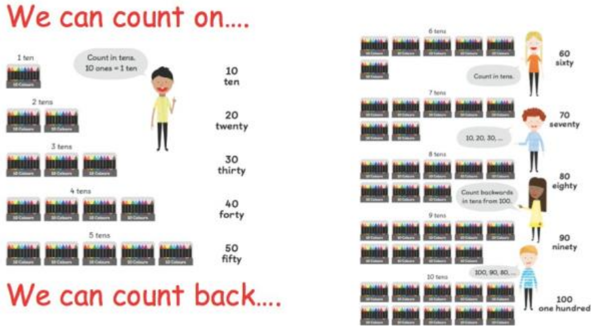
There are 6 cookies and 3 children.
Each child takes one cookie.

Each child takes one more cookie.

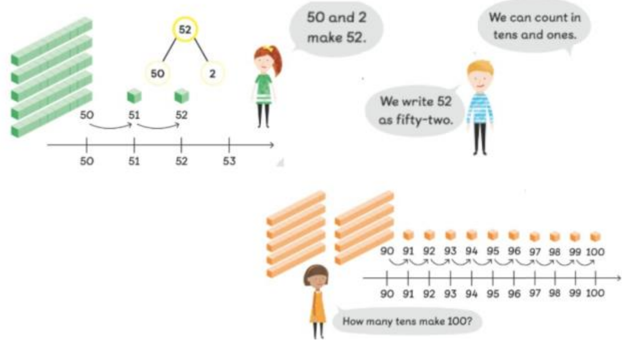
Each child gets 2 cookies.

Year 2
Place Value

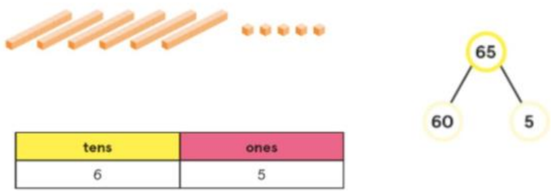
Counting in tens to 100:



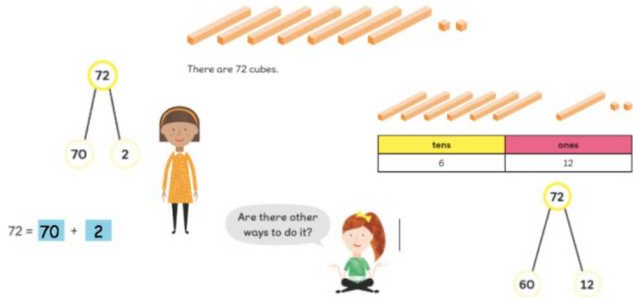
Counting in tens and ones:



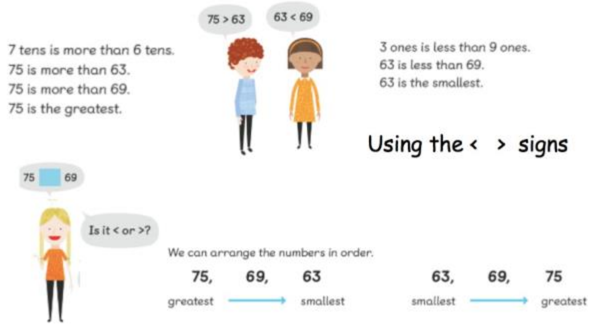
We can represent two-digit numbers in these ways:



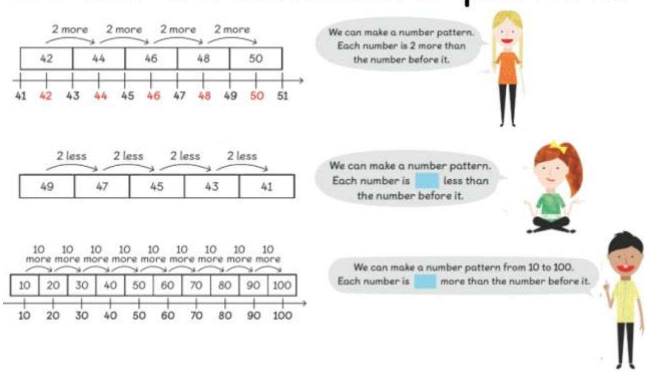
We can make numbers using different number bonds:



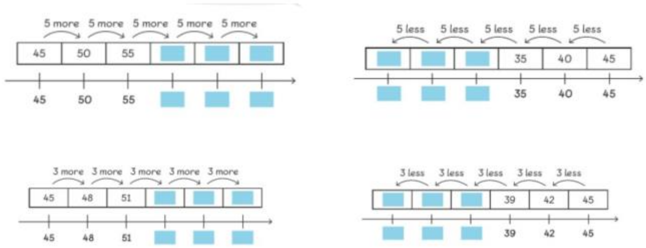
Comparing numbers:



We can extend number patterns:



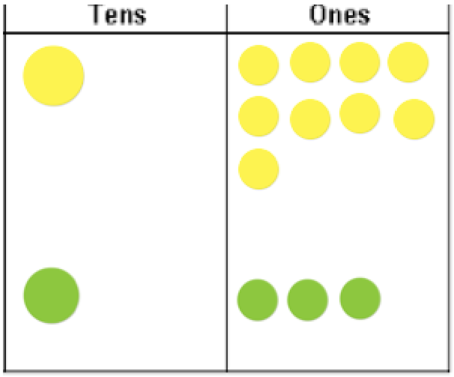
We can find the missing numbers in patterns:



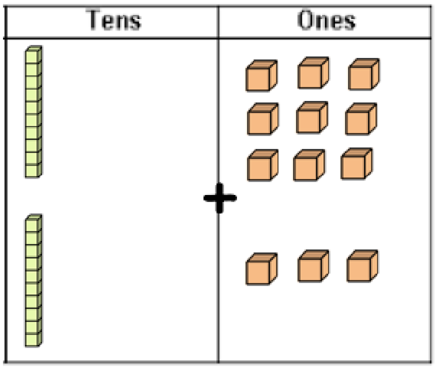
Year 2

Addition

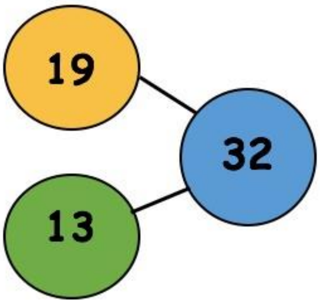
Counters method:



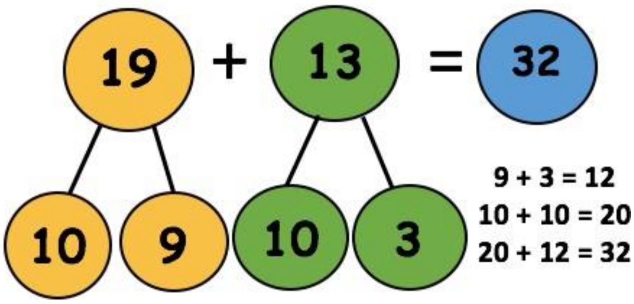
Base 10 method:



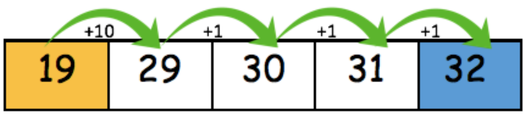
Number bond method:

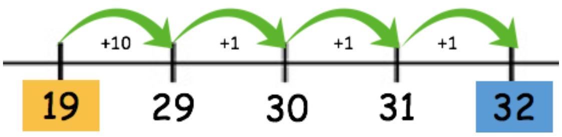


Number bond method:

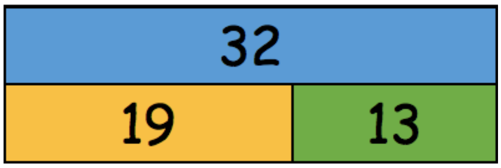


Number line method:





Bar model:



Column addition:

Without renaming:

```
   18
+  11
-----
  2 9
```

With renaming:

```
   19
+  13
-----
   32
   1
```

Expanded method:

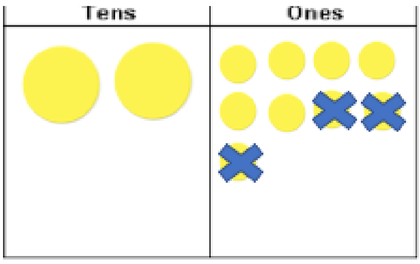
```
  19
 +13
----
  12
  20
----
  32
```

Abstract calculations:

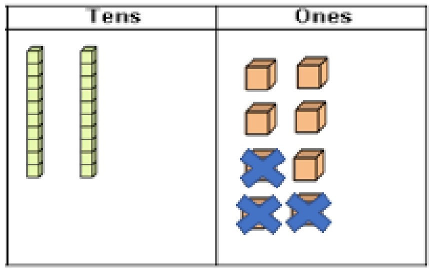
Commutative	Inverse
19 + 13 = 32	32 - 13 = 19
13 + 19 = 32	32 - 19 = 13

Year 2
Subtraction

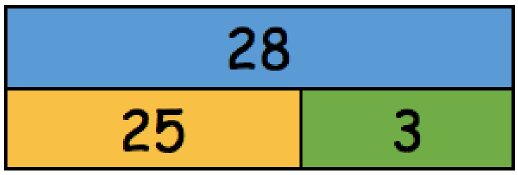
Counters method:



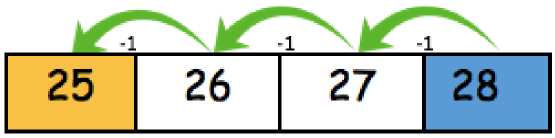
Base 10 method:

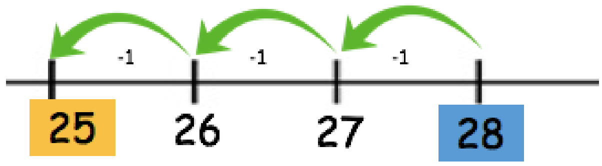


Bar model:

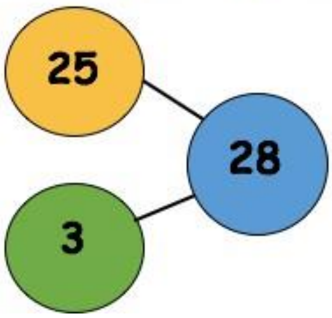


Number line method:

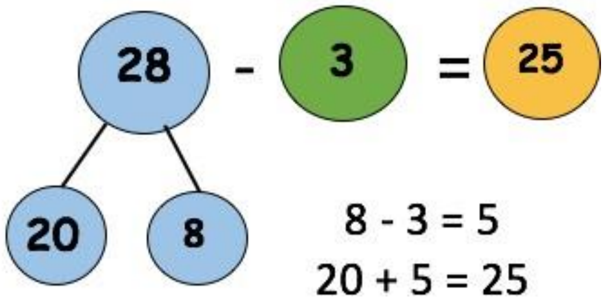




Number bond method:



Number bond method:



8 - 3 = 5
20 + 5 = 25

Column subtraction:

Without renaming:

$$\begin{array}{r} 28 \\ -\ 3 \\ \hline 25 \end{array}$$

With renaming:

$$\begin{array}{r} {}^{1}\cancel{2}{}^{13}\cancel{3} \\ -\ 19 \\ \hline 4 \end{array}$$

Expanded method:

$$\begin{array}{r} 29 \\ -14 \\ \hline 5 \\ 10 \\ \hline 15 \end{array}$$

Abstract calculations:

Commutative	Inverse
25 + 3 = 28	28 - 3 = 25
3 + 25 = 28	28 - 25 = 3

Year 2
Multiplication

Repeated addition:

3 + 3 + 3 + 3 = 12

Number line method:

+3	+3	+3	+3	
0	3	6	9	12

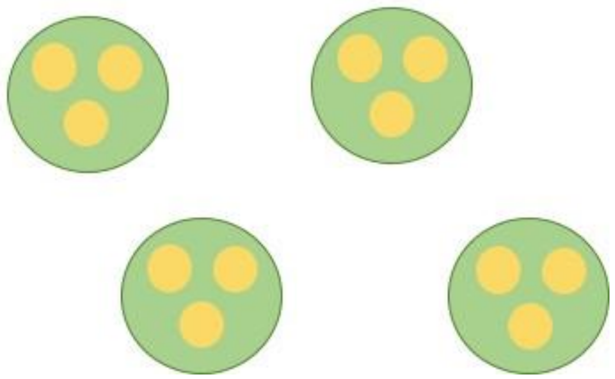
Groups of:

4 groups of 3 is 12

Multiplication:

4 × 3 = 12

Grouping Method:



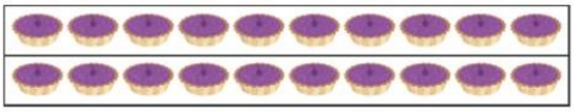
Abstract calculations:

Commutative
3 x 4 = 12
4 x 3 = 12

Year 2
Division

Make a family of multiplication and division facts:

Look at the picture.
Make a family of multiplication and division facts.



2 x 10 = 20 —— 20 ÷ 2 = 10

10 x 2 = 20 —— 20 ÷ 10 = 2

Solving Problems

Ruby has 15 marshmallows.
She packs 5 marshmallows into each bag.
How many bags does Ruby need?

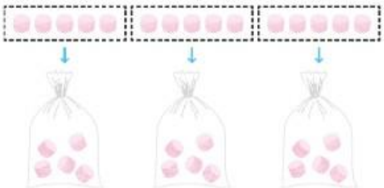
Method 1 Use  to stand for .

Use  for each bag.

Solving Problems:

Ruby has 15 marshmallows.
She packs 5 marshmallows into each bag.
How many bags does Ruby need?

Method 2 Draw a picture.



Solving Problems:

Ruby has 15 marshmallows.
She packs 5 marshmallows into each bag.
How many bags does Ruby need?

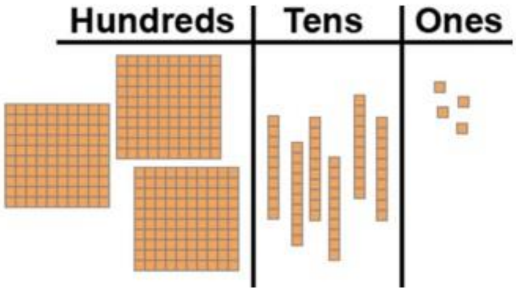
Method 3 Use a division equation.

15 ÷ 5 = 3

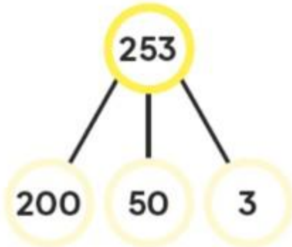
Ruby needs 3 bags.

Year 3
Place Value

Base ten or dienes blocks:



Number bond method:



Value of digits:

hundreds	tens	ones
4	2	7

427 = 4 hundreds + 2 tens + 7 ones
427 = 400 + 20 + 7

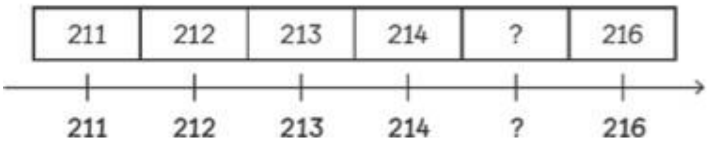
The digit 4 stands for 4 hundreds or 400.
The digit 2 stands for 2 tens or 20.
The digit 7 stands for 7 ones or 7.

We write 427 as four hundred and twenty-seven.

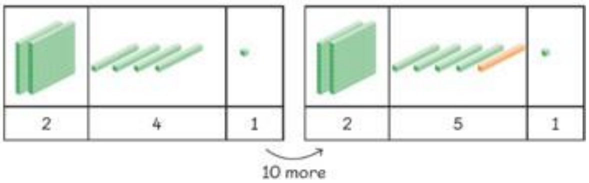
Place value cards:

Separating the numbers apart like this is called **partitioning.**

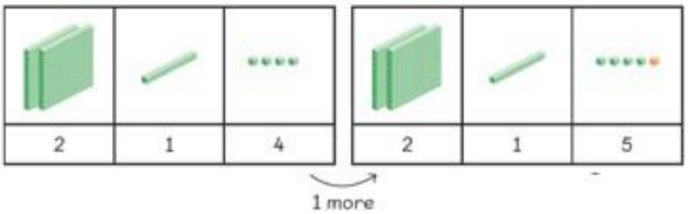
Number lines:



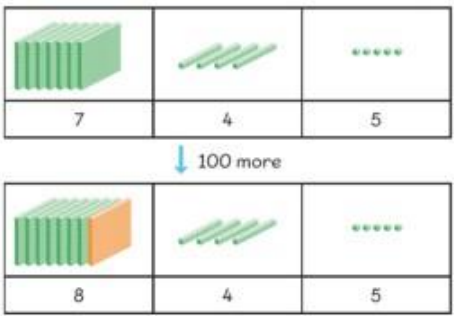
Finding 10 more or less than:



Finding 1 more or less than:

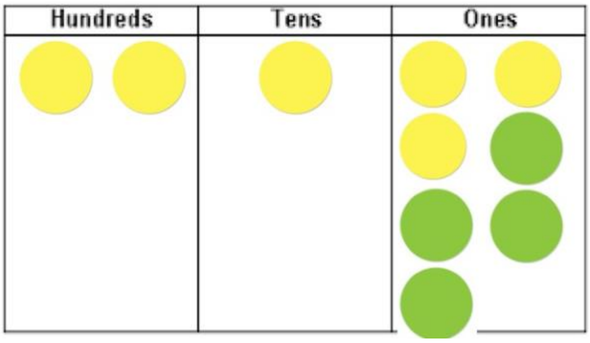


Finding 100 more or less:

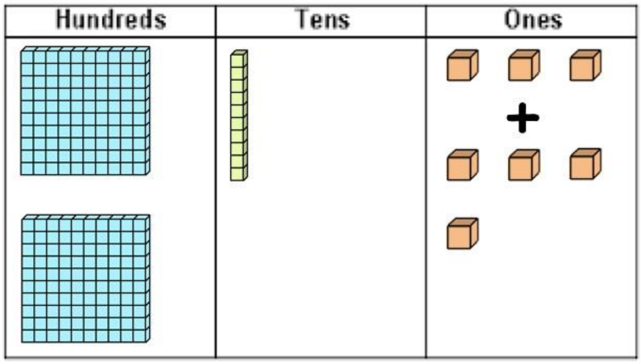


Year 3
Addition

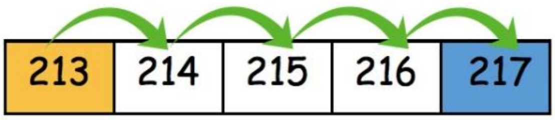
Counters method:

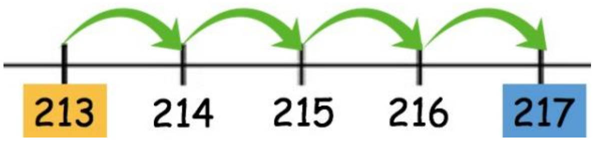


Base 10 method:

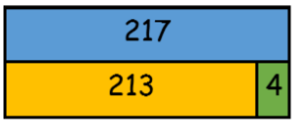


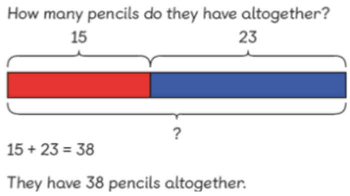
Number line method:



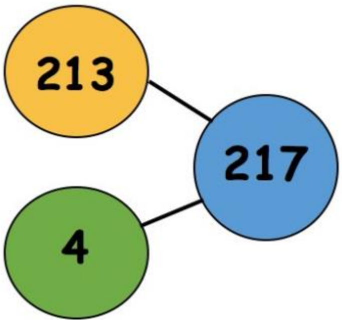


Bar model:

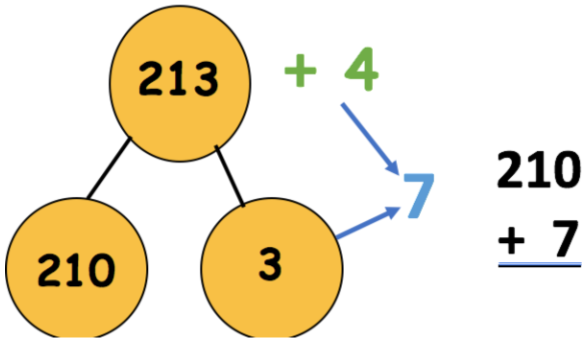




Number bond method:



Number bond method:



Abstract calculations:

Commutative	Inverse
213 + 4 = 217	217 − 4 = 213
4 + 213 = 217	217 − 213 = 4

Column addition:

Without renaming:

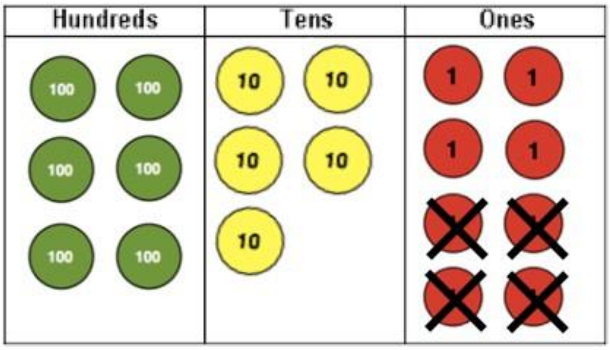
$$\begin{array}{r} 2\ 1\ 3 \\ +\quad\ \ 4 \\ \hline 2\ 1\ 7 \end{array}$$

With renaming:

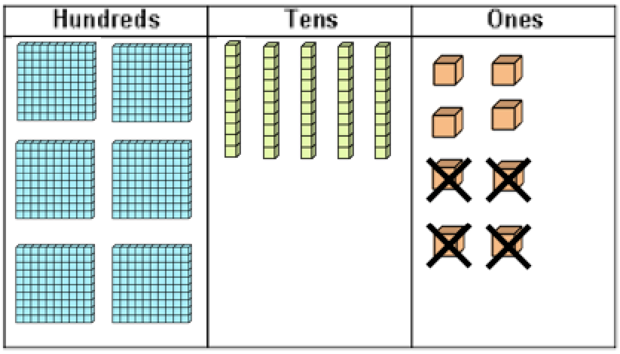
$$\begin{array}{r} {}^{1}\ {}^{1}\ \ \\ 2\ 1\ 3 \\ +\ 4\ 9\ 7 \\ \hline 7\ 1\ 0 \end{array}$$

Year 3
Subtraction

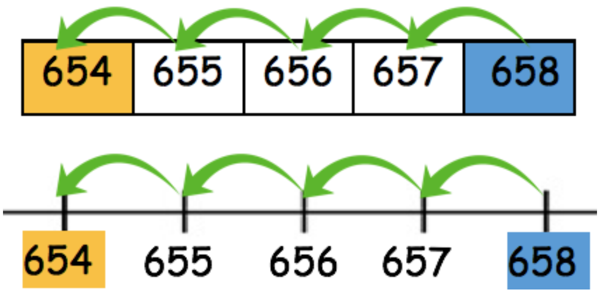
Counters method:



Base 10 method:

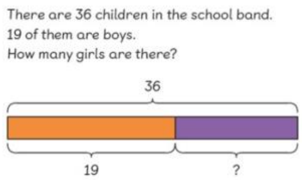


Number line method:

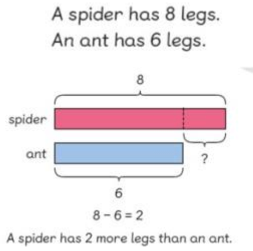


Bar models:

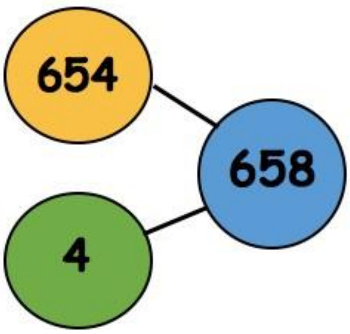
Bar model:



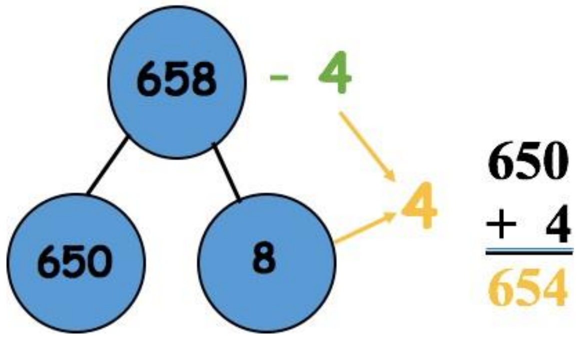
Comparative model:



Number bond method:



Number bond method:



Abstract calculations:

Commutative	Inverse
658 - 4 = 654	654 + 4 = 658
658 - 654 = 4	4 + 654 = 658

Column subtraction:

Without renaming:

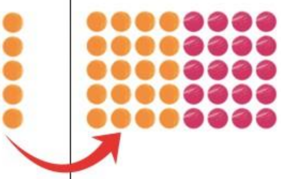
$$\begin{array}{r} 658 \\ -\quad 4 \\ \hline 654 \end{array}$$

With renaming:

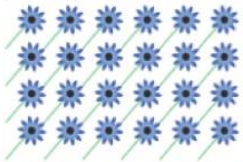
$$\begin{array}{r} 6\,\cancel{5}^{4}\,{}^{1}8 \\ -\ 349 \\ \hline 309 \end{array}$$

Year 3
Multiplication

Arrays:

3 times tables	4 times tables	8 times tables
		
3 x 5 = 15	4 x 5 = 20	8 x 5 = 40 *(doubling the 4 times tables)*

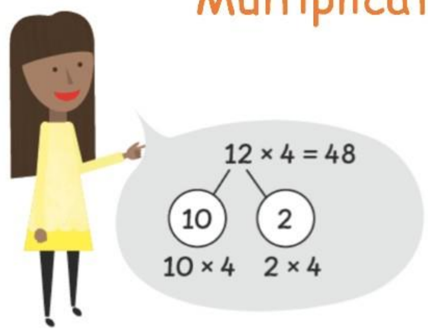
Make a family of multiplication and division facts:



6 × 4 = 24 — 24 ÷ 6 = 4

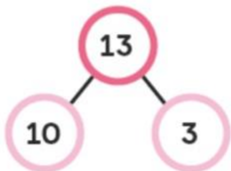
4 × 6 = 24 — 24 ÷ 4 = 6

Number bond strategy: *Multiplication*



Bridged column method: *Without renaming*

13 × 3 = 39



	t	o
	1	3
×		3
		9
+	3	0
	3	9

Bridged column method: *With renaming*

Multiply the ones by 4.

	t	o
	2	5
×		4
	2	0

5 ones × 4 = 20 ones
20 ones = 2 tens

Multiply the tens by 4.

	t	o
	2	5
×		4
	2	0
	8	0

2 tens × 4 = 8 tens

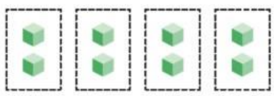
Add the products.

	h	t	o
		2	5
×			4
		2	0
+		8	0
	1	0	0

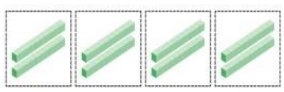
20 + 80 = 100

Short multiplication: *Without renaming*

2 x 4 = 8



2 x 40 = 80



	t	o
	2	0
×		4
	8	0

Short multiplication: *With renaming*

Multiply the ones by 4.

	t	o
2 tens	2	
	4	7
×		4
		8

8 ones

7 ones × 4 = 28 ones
28 ones = 2 tens + 8 ones

Multiply the tens by 4.

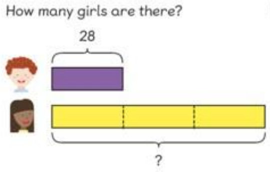
	h	t	o
		2	
		4	7
×			4
	1	8	8

4 tens × 4 = 16 tens
16 tens + 2 tens = 18 tens

Solving word problems: *Bar model*

There are 28 boys in a group.
There are 3 times as many girls as there are boys.

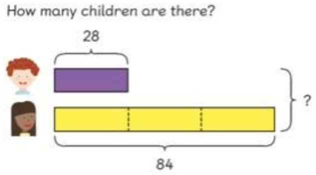
(a) How many girls are there?



28 × 3 = 84

There are 84 girls.

(b) How many children are there?



28 + 84 = 112

There are 112 children altogether.

Year 3
Division

Grouping: 'groups of'

Put 8 into **groups of** 4.

$8 \div 4 = 2$

2 plates are needed.

*"I have made groups of **4**.
There are **2** equal groups.
There are **4** in each group.
2 equal groups of **4** equals **8**."*

Grouping: 'equal groups'

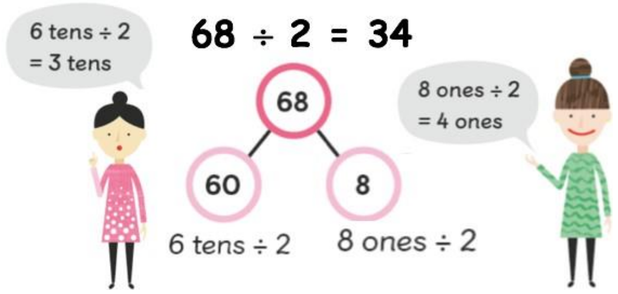
Put 8 into 4 **equal groups**.

$8 \div 2 = 4$

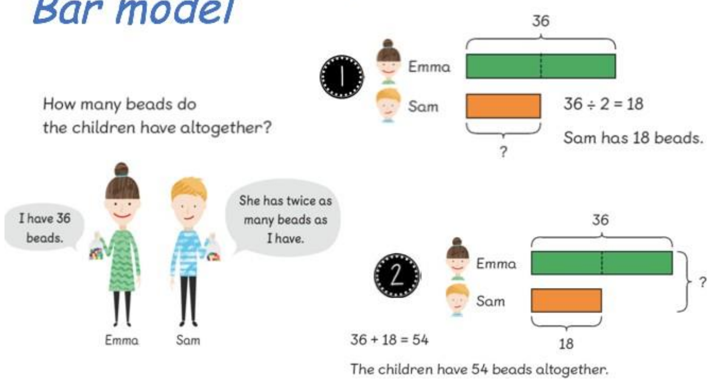
4 trays are needed.

*"There are **4** equal groups.
There are **2** in each group.
4 equal groups of **2** equals **8**."*

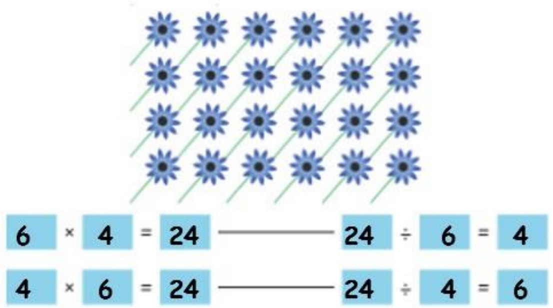
Number bond strategy: Division



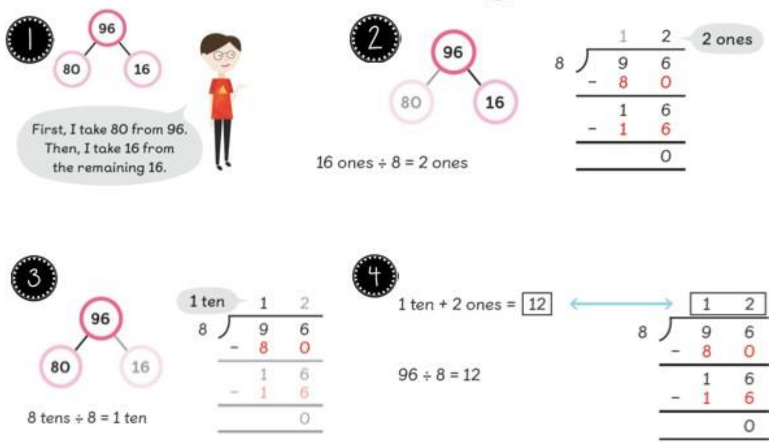
Solving word problems: Bar model



Make a family of multiplication and division facts:

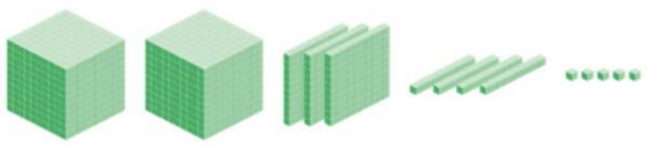


Number bond and long division:



Year 4
Place Value

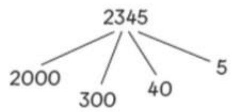
Base ten or dienes blocks:
Thousands/Hundreds/Tens/Ones

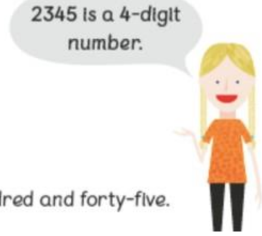


2 thousands + 3 hundreds + 4 tens + 5 ones

Partitioning:

2345 = 2000 + 300 + 40 + 5





We write 2345 as two thousand, three hundred and forty-five.

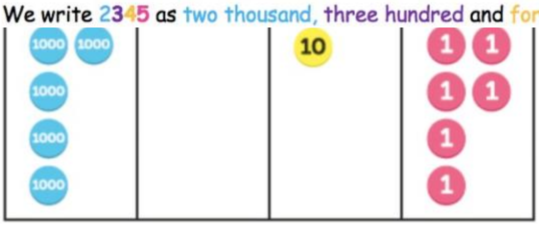
Value of digits:

2 thousands + 3 hundreds + 4 tens + 5 ones

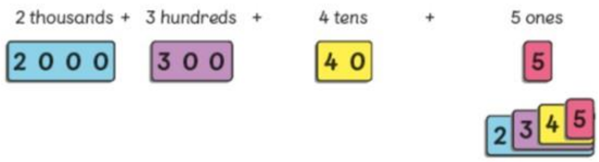
thousands	hundreds	tens	ones
2	3	4	5

2345 = 2 thousands + 3 hundreds + 4 tens + 5 ones
2427 = 2000 + 300 + 40 + 5
The digit 2 stands for 2 thousand or 2000.
The digit 3 stands for 3 hundreds or 300.
The digit 4 stands for 4 tens or 40.
The digit 5 stands for 5 ones or 5.

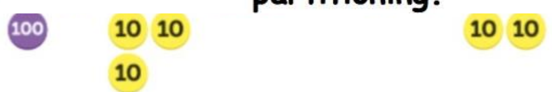
We write 2345 as two thousand, three hundred and forty-five.



Place value cards:

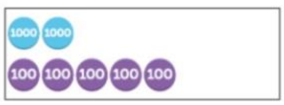


Separating the numbers like this is called **partitioning.**

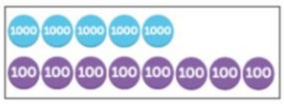


352 is more than 241
352 is greater than 241
352 > 241

Comparing numbers:



thousands	hundreds	tens	ones
2	5	0	0



thousands	hundreds	tens	ones
5	8	0	0

2500 is less than 5800.
2500 < 5800

2500 is less than 5800
2500 < 5800

Number patterns:

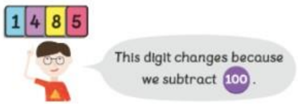
What number is 1 more than 1485?

1485 + 1 = 1486

What number is 10 more than 1485?

1485 + 10 = 1495

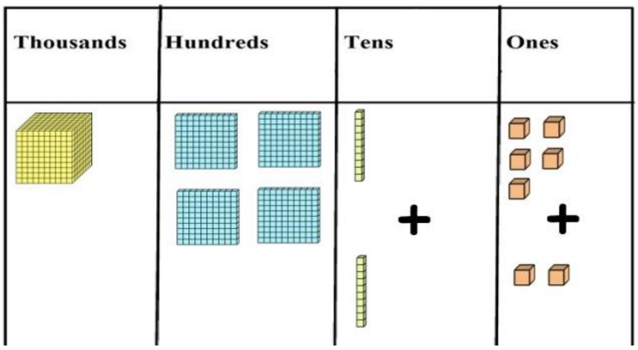
What number is 100 less than 1485?



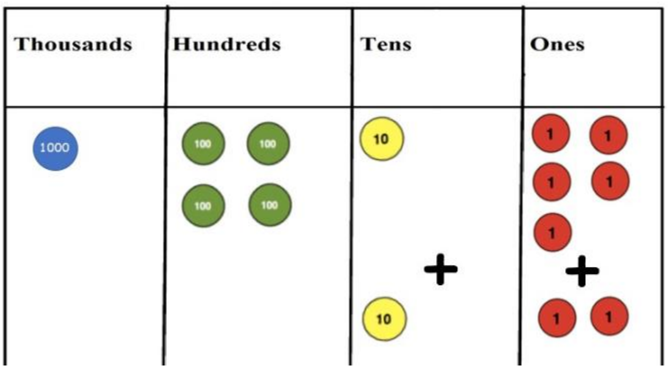
1485 - 100 = 1395

Year 4
Addition

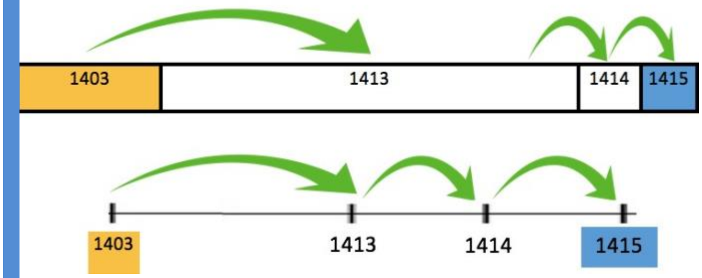
Base 10 method:



Counters method:



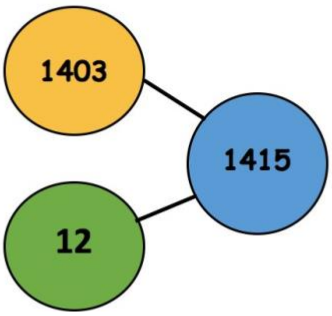
Number line method:



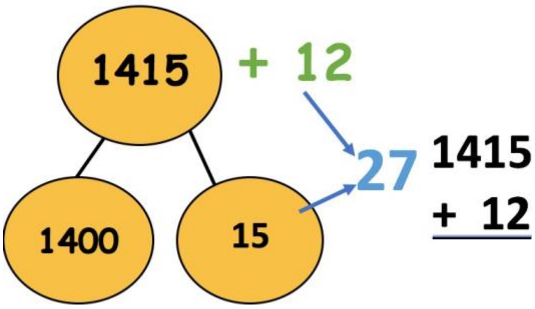
Abstract calculations:

Commutative	Inverse
1415 + 12 = 1427	1427 – 12 = 1415
12 + 1415 = 1427	1427 – 1415 = 12

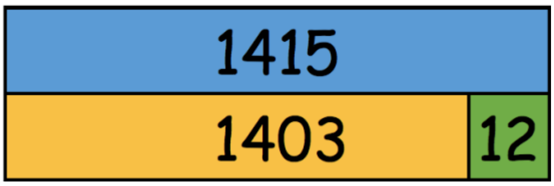
Number bond method:



Number bond method:



Bar model:



Column addition:

Without renaming:

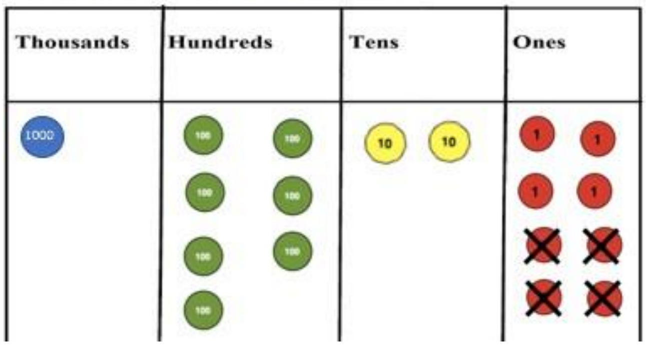
```
  1 4 1 5
+     1 2
---------
  1 4 2 7
```

With renaming:

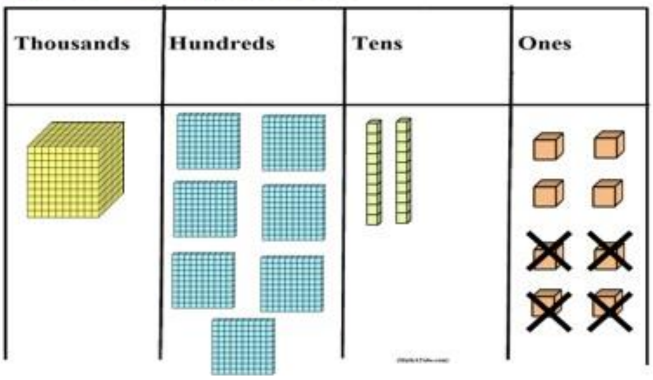
```
    1   1
  1 4 1 5
+     9 6
---------
  1 5 1 1
```

Year 4
Subtraction

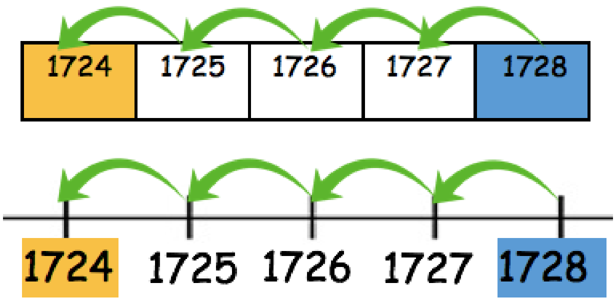
Counters method:



Base 10 method:



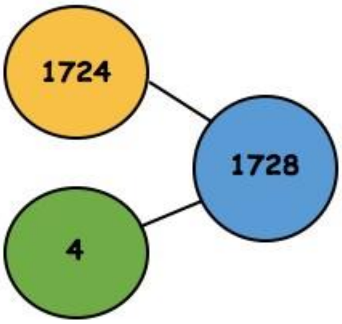
Number line method:



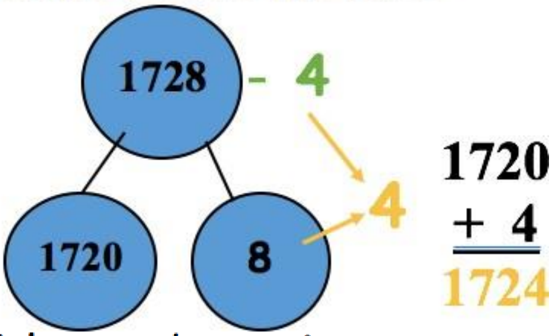
Abstract calculations:

Commutative	Inverse
1728 - 4 = 1724	1724 + 4 = 1728
1728 - 1724 = 4	4 + 1724 = 1728

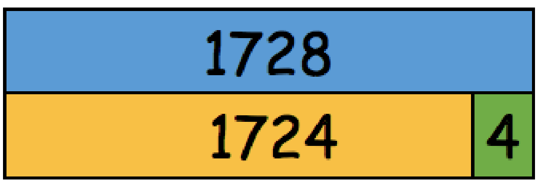
Number bond method:



Number bond method:



Bar model:



Column subtraction:

Without renaming:

```
  1 7 2 8
-       4
  1 7 2 4
```

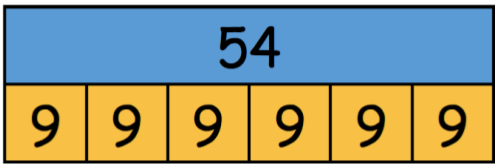
With renaming:

```
    6 11 18
  1 7  2  8
-   3  4  9
    3  7  9
```

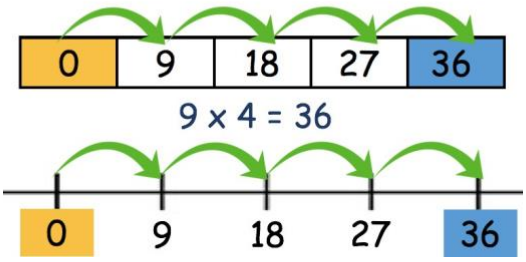
Year 4
Multiplication

Bar model:

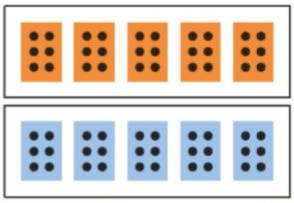
$6 \times 9 = 54$



Number line method:

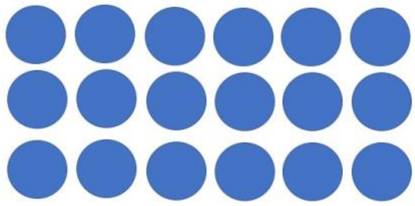


Multiply 3 numbers:



$2 \times 5 \times 6 = 10 \times 6 = 60$

Array method:



$6 \times 3 = 18$ OR $3 \times 6 = 18$

Multiplying by 10:

What is the product of 9 and 30?

$9 \times 30 = \square$

Method 1

$$\begin{array}{r} 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ +\ 30 \\ \hline \\ \hline \end{array}$$

Method 2

$9 \times 3 = 27$

9×3 tens = 27 tens

$9 \times 30 = 270$

Method 3

$9 \times 30 = 9 \times 3 \times 10$

$= 9 \times 3 \times 10$

$= 27 \times 10$

= 27 tens

= 270

Multiplying by 100:

$7 \times 300 = \square$

Method 1

$$\begin{array}{r} 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ +\ 300 \\ \hline 2100 \\ \hline \end{array}$$

Method 2

$7 \times 3 = 21$

7×3 hundreds = 21 hundreds

$7 \times 300 = 2100$

21 hundreds = 2100

Method 3

$7 \times 300 = 7 \times 3 \times 100$

$= 7 \times 3 \times 100$

$= 21 \times 100$

= 21 hundred

= 2100

Bridged and short multiplication:

$$\begin{array}{rrrr} & & 2 & 3 \\ \times & & & 6 \\ \hline & & 1 & 8 \\ + & 1 & 2 & 0 \\ \hline & 1 & 3 & 8 \\ \hline \end{array} \qquad \begin{array}{rrrr} & & \overset{1}{2} & 3 \\ \times & & & 6 \\ \hline & 1 & 3 & 8 \\ \hline \end{array}$$

2 digit x 1 digit

Bridged and short multiplication:

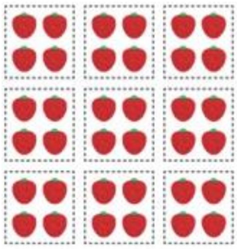
$$\begin{array}{rrrr} & \overset{1}{4} & 7 & 3 \\ \times & & & 2 \\ \hline & 9 & 4 & 6 \\ \hline \end{array}$$

3 digit x 1 digit

Year 4
Division

Division by grouping:

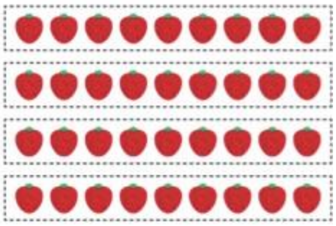
Placing into 9 equal groups



$36 \div 9 = 4$

Each group has 4 strawberries.

Placing in groups of 9

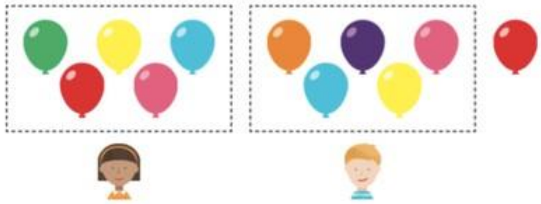


$36 \div 9 = 4$

There are 4 groups.

Grouping with remainders:

There were 11 balloons.



$11 \div 2 = 5$ remainder 1

The quotient is 5 and the remainder is 1.
Each friend got 5 balloons.
There was 1 balloon left over.

Divide using multiplication:

$$24 \div 3 = \underline{8}$$

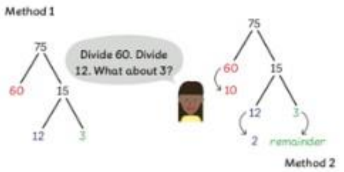
$$3 \times \underline{8} = 24$$

Dividing by 1, 10 and 100:

$4 \div 4 = \square$ $\quad 40 \div 4 = \square$ $\quad 400 \div 4 = \square$

$4 \div 4 = 1$ $\quad 40 \div 4 = 10$ $\quad 400 \div 4 = 100$

Divide with remainders:



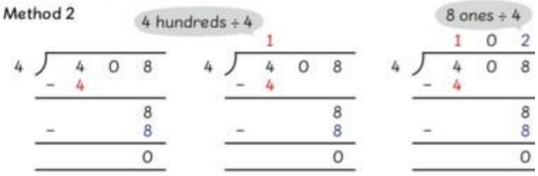
Part-part-whole method

Divide without remainders:

Part-part-whole method

Method 2

Long division

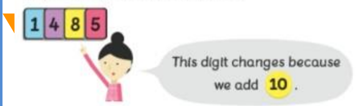


Number patterns:

What number is 1 more than 1485?

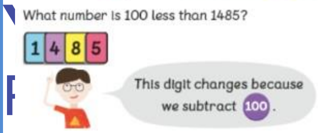
1485 + 1 = 1486

What number is 10 more than 1485?



1485 + 10 = 1495

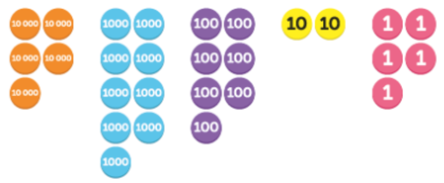
What number is 100 less than 1485?



1485 - 100 = 1395

Value of digits / place value counters. place value charts

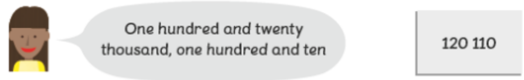
Show 59 725 using number discs.



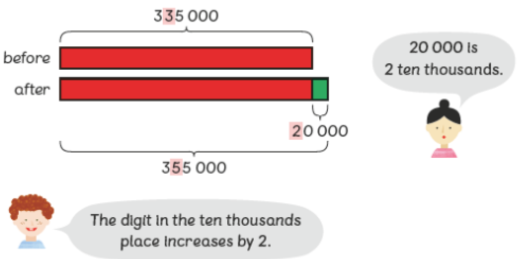
ten thousands	thousands	hundreds	tens	ones
5	9	7	2	5

Reading numbers – place value charts

hundred thousands	ten thousands	thousands	hundreds	tens	ones
1	1 1		1	1	

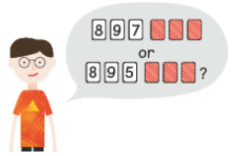


Comparing Numbers

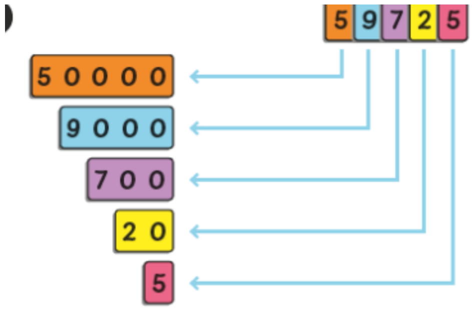


Who made a smaller number?

☐ is greater than ☐.
☐ is less than ☐.



Reading numbers – place value cards



The digit 5 is in the ten thousands place.
It stands for 50 thousands or 50 000.

The digit 9 is in the thousands place.
It stands for 9 thousands or 9000.

The digit 7 is in the hundreds place.
It stands for 7 hundreds or 700.

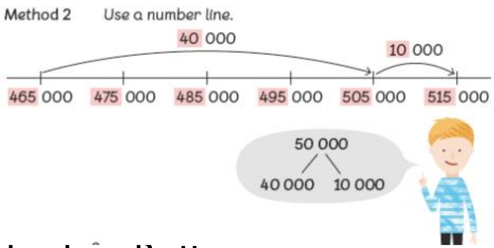
The digit 2 is in the tens place.
It stands for 2 tens or 20.

59 725 = 50 000 + 9000 + 700 + 20 + 5

Com

We read 59 725 as fifty-nine thousand, seven hundred and twenty-five.

Method 2 Use a number line.

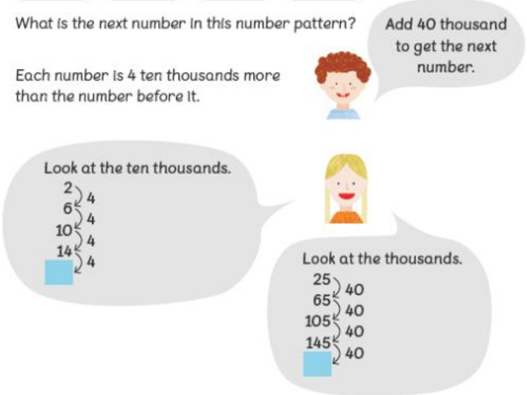


Number Patterns

25 160, 65 160, 105 160, 145 160, ...

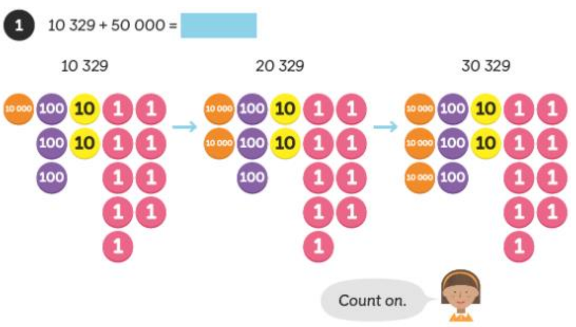
What is the next number in this number pattern?

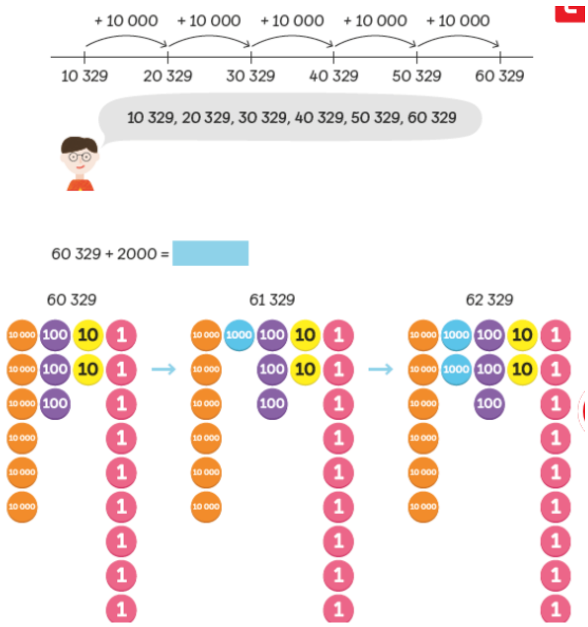
Each number is 4 ten thousands more than the number before it.



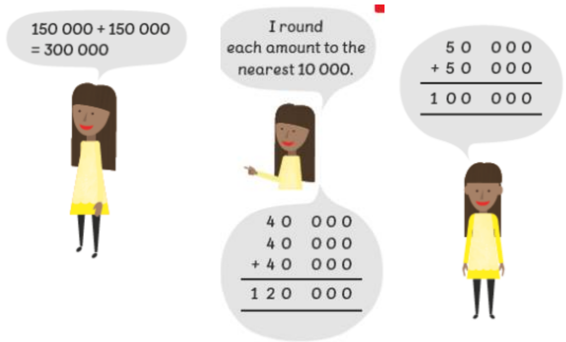
Year 5
Addition

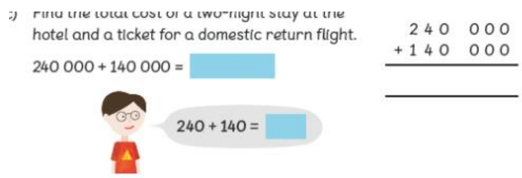
Count on to add





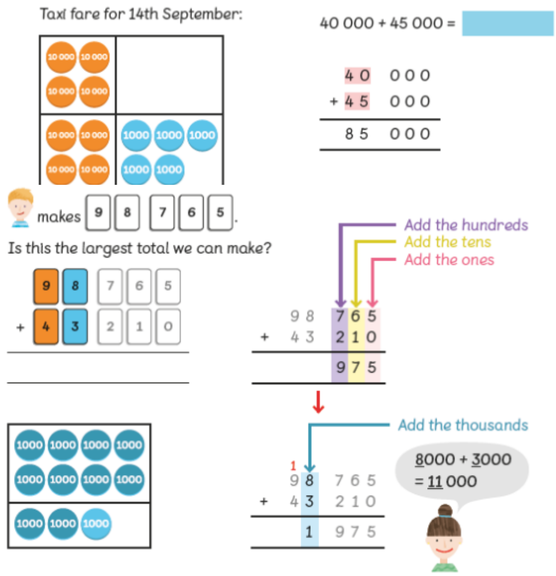
Mental Calculation



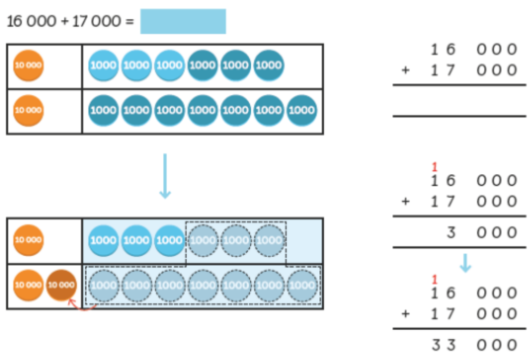


Column Method

Place value counters



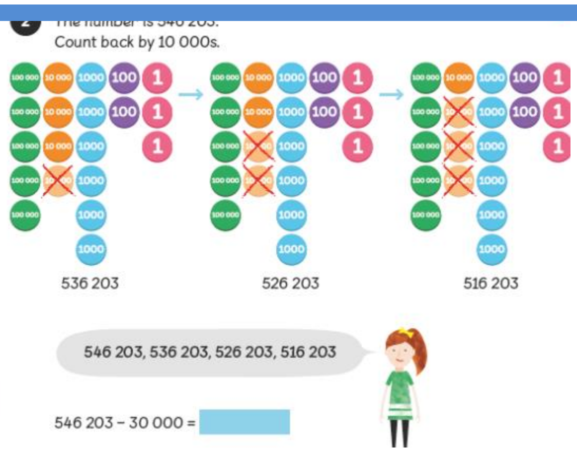
with renaming

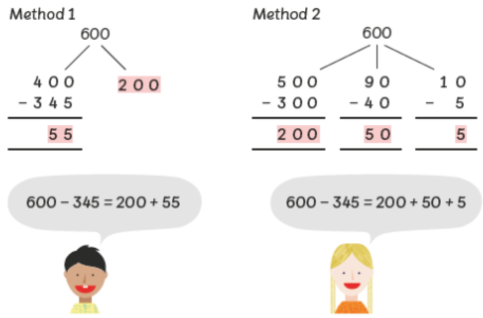


Y
Subtraction

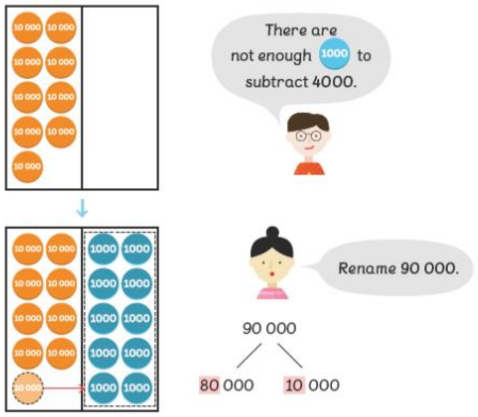
Count back

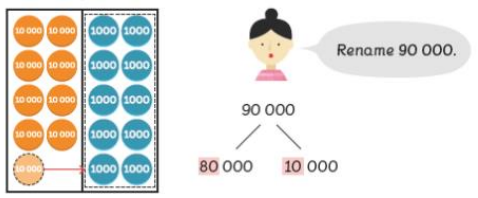
Subtraction using partitioning

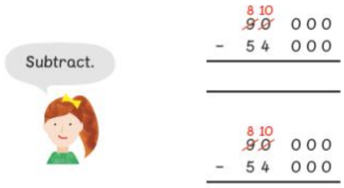




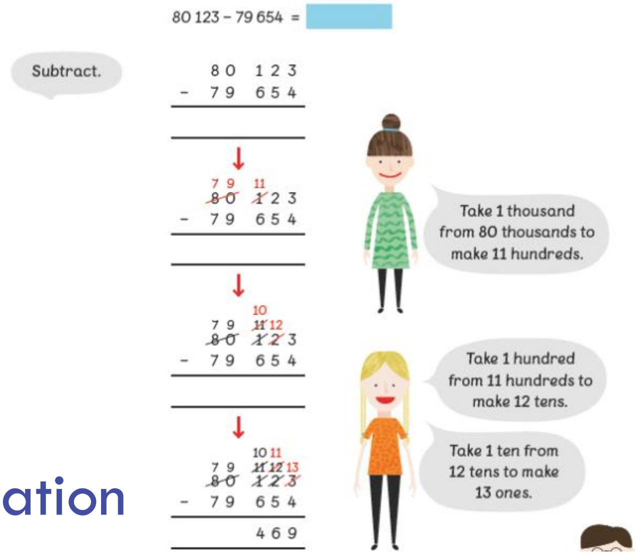
Subtraction with renaming

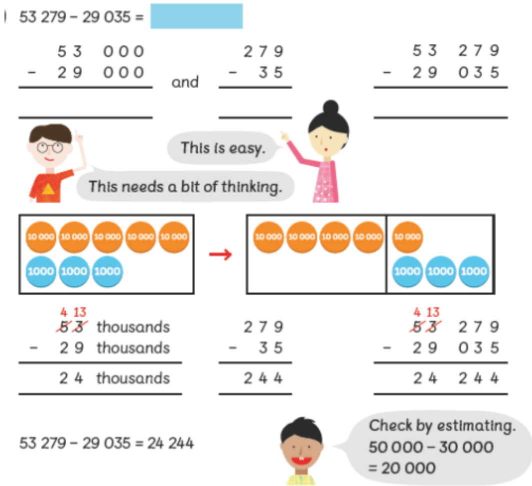






Col



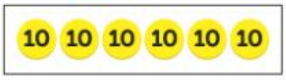


Year 5
Multiplication

Multiplying by 10 100 and 1000

Using Place value counters

1 $6 \times 10 =$



$6 \times 10 = 6 \times 1$ ten
$= 6$ tens

2 $6 \times 100 =$

$6 \times 100 = 6 \times 1$ hundred
$= 6$ hundreds

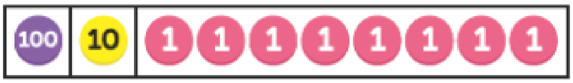
3 $6 \times 1000 =$

$6 \times 1000 = 6 \times 1$ thousand
$= 6$ thousands

Multiply 3 digit by 1 digit and 4 digit by 1 digit

$3 \times £118 =$

This shows 118.



3

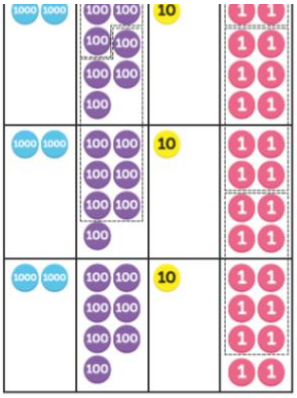
$3 \times 8 = 24$
$3 \times 10 = 30$
$3 \times 100 = 300$
$3 \times 118 = 354$

$$\begin{array}{r} 1\,1\,8 \\ \times \quad 3 \\ \hline 2\,4 \\ 3\,0 \\ +\,3\,0\,0 \\ \hline 3\,5\,4 \end{array}$$

2 4 → multiply by ones
3 0 → multiply by tens
3 0 0 → multiply by hundreds

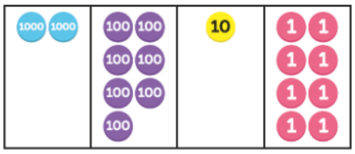
Three sets of 8 boxes cost £354.

$$\begin{array}{r} 2\,7\,1\,8 \\ \times \quad 4 \\ \hline 7\,2 \end{array} \rightarrow \begin{array}{r} 2\,7\,1\,8 \\ \times \quad 4 \\ \hline 8\,7\,2 \end{array} \rightarrow \begin{array}{r} 2\,7\,1\,8 \\ \times \quad 4 \\ \hline 1\,0\,8\,7\,2 \end{array}$$



1 $2718 \times 4 =$

$2000 \times 4 = 8\,000$
$700 \times 4 = 2\,800$
$10 \times 4 = 40$
$8 \times 4 = 32$
$2718 \times 4 = 10\,872$

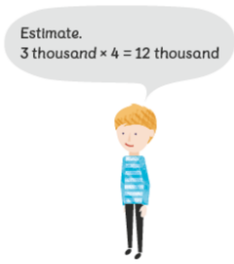


2 thousands → 8 thousands
7 hundreds → 28 hundreds
1 ten → 4 tens
8 ones → 32 ones

2 $2718 \times 4 =$

$$\begin{array}{r} 2\,7\,1\,8 \\ \times \quad 4 \\ \hline 3\,2 \\ 4\,0 \\ 2\,8\,0\,0 \\ +\,8\,0\,0\,0 \\ \hline 1\,0\,8\,7\,2 \end{array}$$

$2718 \times 4 = 10\,872$



Using partitioning

$1022 \times 4 =$

$1000 \times 4 = 4000$
$20 \times 4 = 80$
$2 \times 4 = 8$
$1022 \times 4 = 4088$

$$\begin{array}{r} 1\,0\,2\,2 \\ \times \quad 4 \\ \hline 8 \\ 8\,0 \\ 0 \\ +\,4\,0\,0\,0 \\ \hline 4\,0\,8\,8 \end{array}$$

8 → multiply by ones
8 0 → multiply by tens
0 → multiply by hundreds
4 0 0 0 → multiply by thousands

4 $2718 \times 4 =$ ☐

Double 2718

$$\begin{array}{r} {\scriptstyle 1\ \ \ 1\ \ } \\ 2\,7\,1\,8 \\ +\ 2\,7\,1\,8 \\ \hline 5\,4\,3\,6 \\ \hline \end{array}$$

Double 5436

$$\begin{array}{r} {\scriptstyle 1\ \ } \\ 5\,4\,3\,6 \\ +\ 5\,4\,3\,6 \\ \hline 1\,0\,8\,7\,2 \\ \hline \end{array}$$

Using PV counters for 2digit by 2 digit

1 $14 \times 12 =$ ☐

10	1 1

14×10
$= 14 \times 1$ ten
$= 14$ tens

140

$14 \times 2 = 28$

$$\begin{array}{r} 14 \times 10 = 140 \\ 14 \times \ \ 2 = \ \ 28 \\ \hline 14 \times 12 = 168 \\ \hline \end{array}$$

$14 \times 12 = 168$

2 $14 \times 22 =$ ☐

10 10	1 1

14×20
$= 14 \times 2$ tens
$= 28$ tens

280

$14 \times 2 = 28$

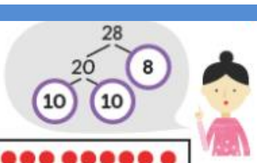
$$\begin{array}{r} 14 \times 20 = 280 \\ 14 \times \ \ 2 = \ \ 28 \\ \hline 14 \times 22 = 308 \\ \hline \end{array}$$

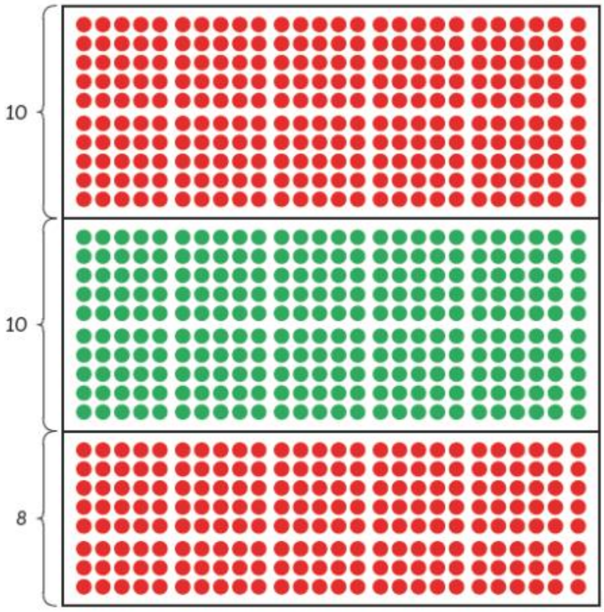
$14 \times 22 = 308$

Partitioning 2d by 2d (grid method)

2 There are 28 rows.
Each row consists of 26 seats.

28 × 26 = ☐



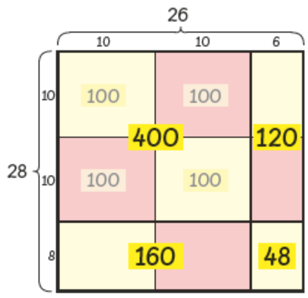


10 × 26 = 260
10 × 26 = 260
8 × 26 = 208
28 × 26 = 728

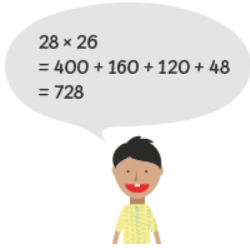
$$\begin{array}{r} \scriptstyle 4 \\ 2\,6 \\ \times \quad 8 \\ \hline 2\,0\,8 \\ \hline \end{array}$$

There are 728 seats.

1 There are 28 rows.
Each row consists of 26 seats.

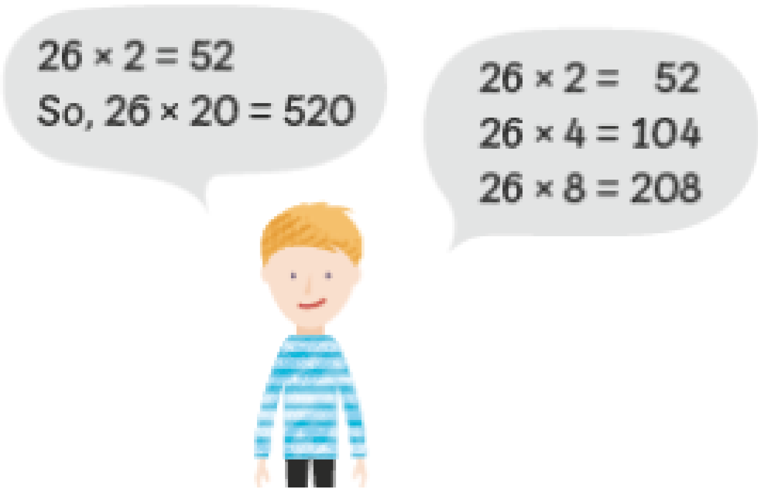


There are 728 seats.



3 28 × 26 = ☐

20 × 26 = 520
8 × 26 = 208
28 × 26 = 728



Formal written method 2d by 2d

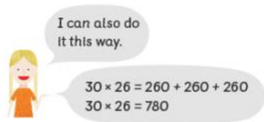
Using known facts

4 28 × 26 = ☐

$$\begin{array}{r} \scriptstyle 4 \\ 2\,8 \\ \times\ 2\,6 \\ \hline 8 \end{array} \longrightarrow \begin{array}{r} \scriptstyle 4 \\ 2\,8 \\ \times\ 2\,6 \\ \hline 1\,6\,8 \end{array}$$

$$\begin{array}{r} \scriptstyle 1 \\ 2\,8 \end{array} \qquad \begin{array}{r} \scriptstyle 1 \\ 2\,8 \end{array}$$

5 28 × 26 = ☐



28 × 26 = 780 − ☐

Can you continue her method?

Partitioning 3d by 2d

12 × 132 =

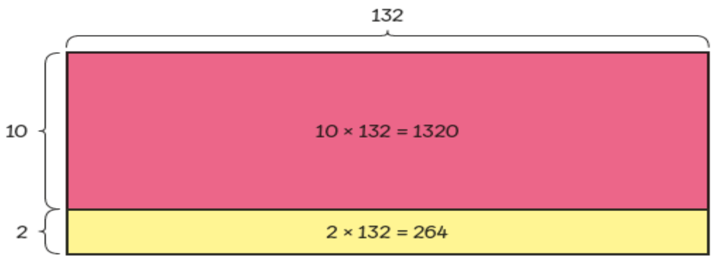
2 times	200	60	4	264
10 times	1000	300	20	1320

12 × 132 = 1320 + 264
= 1584

It costs about 1584 Hong Kong dollars.

Grid method 3d by 2d

12 × 132 =



12 × 132 = 1320 + 264
= 1584

Formal written method

3 12 × 132 =

$$
\begin{array}{r l}
1\,3\,2 & \\
\times \quad 1\,2 & \\
\hline
2\,6\,4 & \rightarrow \text{multiply by 2} \\
+\,1\,3\,2\,0 & \rightarrow \text{multiply by 10} \\
\hline
1\,5\,8\,4 & \\
\hline
\end{array}
$$

Year 5
Division

Dividing by 10, 100 and 1000

Partitioning

How many can we get from 4792?

contains 1000 pieces.

How many 1000s in 4000?

4792

$\boxed{4000}$ 792

Here is the remainder.

There are 4 in 4000.

$4000 \div 1000 = 4$

4 thousands ÷ 1 thousand = 4

There are 4 groups of 1000 in 4000.

2 How many can we get from 4792?

contains 100 pieces.

4792

4000 700 $\boxed{92}$

How many 100s in 4700?

Here's the remainder.

$4700 \div 100 = 47$

47 hundreds ÷ 1 hundred = 47

There are 47 groups of 100 in 4700.

3 How many can we get from 4792?

contains 10 pieces.

4792

4000 700 90 $\boxed{2}$

How many 10s in 4790?

Here's the re

$4790 \div 10 = 479$

479 tens ÷ 1 ten = 479

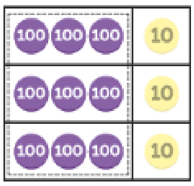
There are 479 groups of 10 in 4790.

2 $930 \div 3 =$ ____

930

900 ____

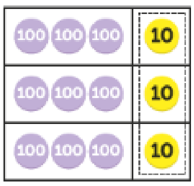
$$\begin{array}{r|rrr} 3 & 9 & 3 & 0 \\ - & 9 & 0 & 0 \\ \hline \end{array}$$



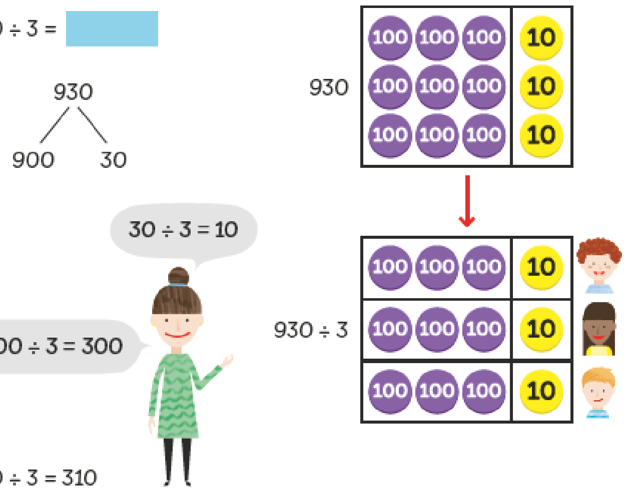
930

900 30

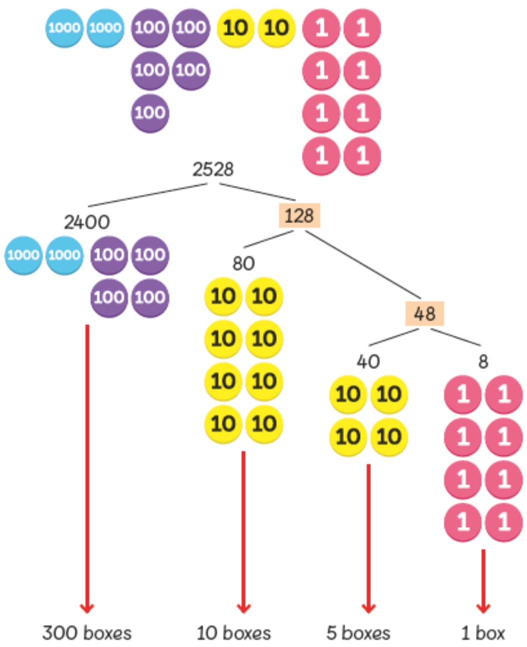
$$\begin{array}{r|rrr} 3 & 9 & 3 & 0 \\ - & 9 & 0 & 0 \\ \hline & & 3 & 0 \\ - & & 3 & 0 \\ \hline & & & 0 \\ \hline \end{array}$$



930 ÷ 3 =



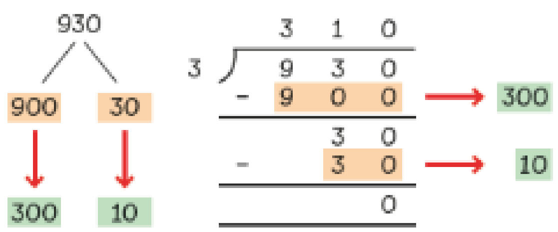
930 ÷ 3 = 310



2528 ÷ 8 = 300 + 10 + 5 + 1
= 316

316 boxes are needed.

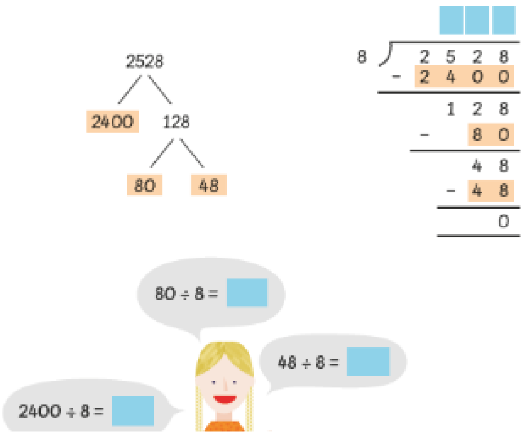
Abstract written methods



2 's story

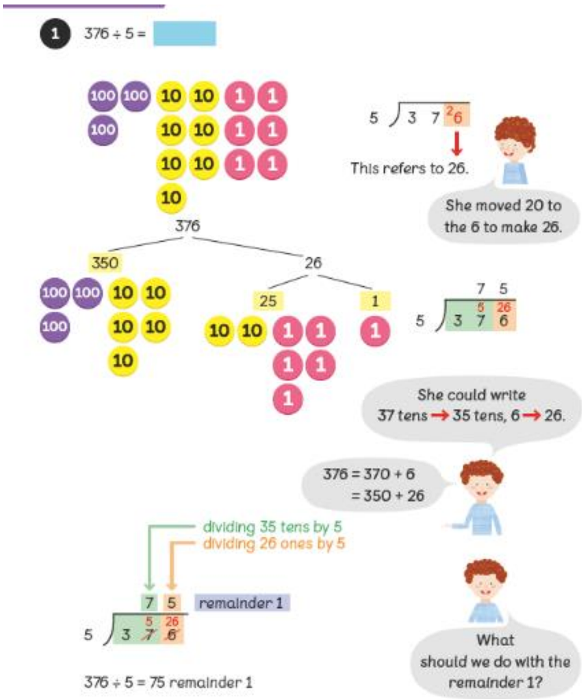
2528 ml of juice is put into 8 containers so that each container holds the same volume. What is the volume of juice in each container?

2528 ml ÷ 8 =



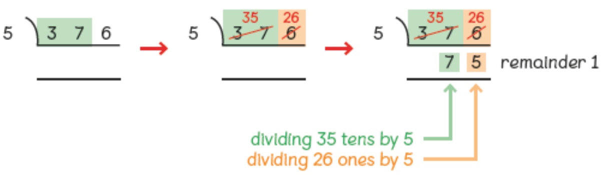
Dividing

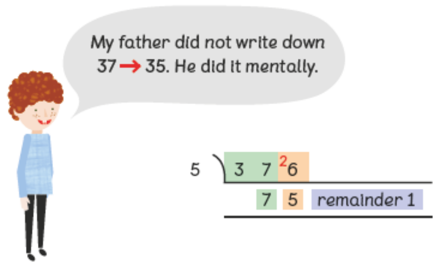
Using PV counters



With written methods

2 $376 \div 5 =$ ☐





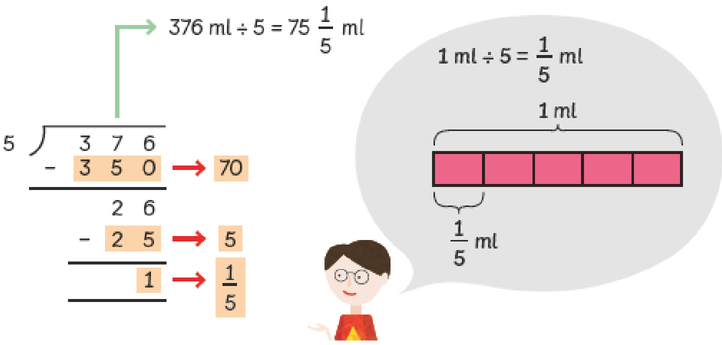
3 376 children in a school are put into 5 equal groups.
Is this possible?

$376 \div 5 = 75$ remainder 1

It is not possible.
There will always be one child left over, who does not belong to any group.

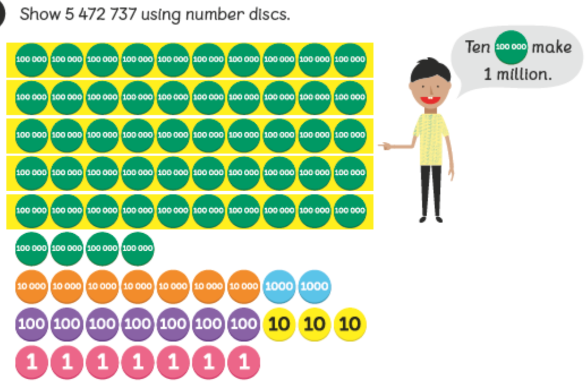
As a fraction

4 376 ml of liquid soap is poured into 5 bottles.
Each bottle contains the same amount of soap.
Find the volume of soap in each bottle.

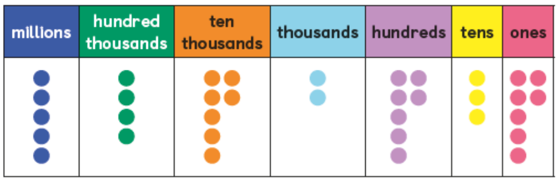


Year 6
Place Value
With counters and PV cards

1 Show 5 472 737 using number discs.

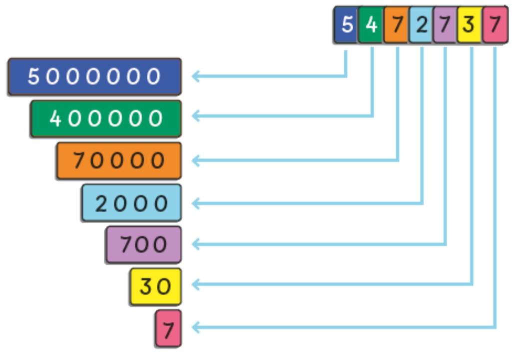


2 Show 5 472 737 on a place-value chart.



millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
5	4	7	2	7	3	7

3 5 472 737 = ▭ + ▭ + ▭ + ▭ + ▭ + ▭ + ▭



Using vocabulary

The digit 5 is in the millions place.
It stands for 5 millions or 5 000 000.

The digit 4 is in the hundred thousands place.
It stands for 4 hundred thousands or 400 000.

The digit 7 appears more than once.
7 is in the ten thousands place.
It stands for 70 000.

7 is also in the hundreds place.
It stands for 700.

7 is also in the ones place.
It stands for 7.

The digit 2 is in the thousands place.
It stands for 2000.

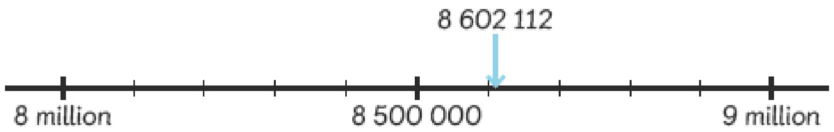
The digit 3 is in the tens place.
It stands for 30.

5 472 737 = 5 000 000 + 400 000 + 70 000 + 2000 + 700 + 30 + 7

We write 5 472 737 as five million, four hundred and seventy-two thousand, seven hundred and thirty-seven.

Rounding using numberlines

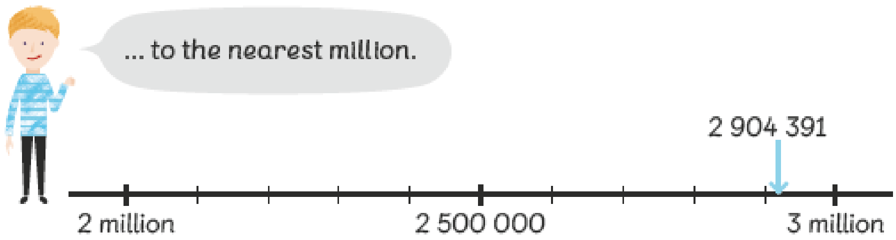
2 Round 8 602 112 to the nearest million.



8 602 112 is closer to 9 million than to 8 million.

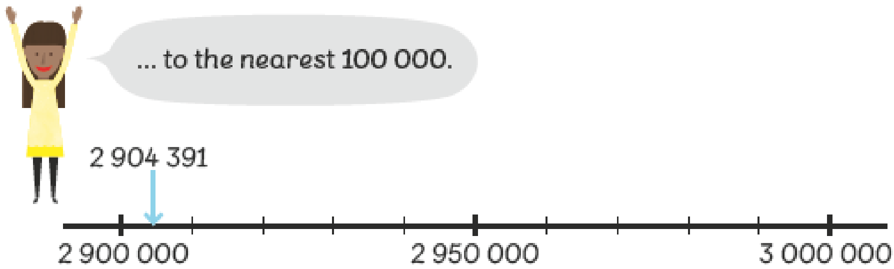
8 602 112 ≈ 9 million

3 Round 2 904 391...



2 904 391 is closer to 3 million than to 2 million.

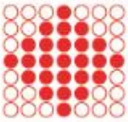
2 904 391 ≈ 3 million



2 904 391 is closer to 2 900 000 than to 3 000 000.

2 904 391 ≈ 2 900 000 (to the nearest 100 000)

2



 1 + 2 + 3

 7 × 7

This expression describes the method.

$7 \times 7 - 4 \times (1 + 2 + 3) =$ ▭

Step 1: Perform the calculation in ( ).
Step 2: Multiply.
Step 3: Subtract.

$1 + 2 + 3 = 6$
$7 \times 7 = 49$
$4 \times 6 = 24$
$49 - 24 = 25$

Are there other ways to express the number of ● in the diagram? Write expressions to describe these ways.

1 wrote this expression:
$6 + 5 - 1 - 2 - 3 - 4$

When there are only + and −, calculate from left to right.

$6 + 5 - 1 - 2 - 3 - 4 = 1$

$6 + 5 = 11$

2 wrote this expression:
$3 \times 4 + 2 - (6 + 5 + 1)$

Perform the calculations in ( ) first.

$3 \times 4 + 2 - (6 + 5 + 1)$
$= 3 \times 4 + 2 - 12$
$= 12 + 2 - 12$
$= 14 - 12$
$= 2$

For + and −, calculate from left to right.

Multiply before adding.

$3 \times 4 + 2 - (6 + 5 + 1) = 2$

3 wrote this expression:
$2 \times 3 \times 6 \div 4 - 5 - 1$

For × and ÷, calculate from left to right.

$2 \times 3 \times 6 \div 4 - 5 - 1$
$= 6 \times 6 \div 4 - 5 - 1$
$= 36 \div 4 - 5 - 1$
$= 9 - 5 - 1$
$= 3$

Subtract from left to right.

$2 \times 3 \times 6 \div 4 - 5 - 1 = 3$

Can you make an expression that has the value of 4? How about the values of 5 or 6?

4 made a different expression that has the value of 3.
$(1 + 2) \div 3 \times 4 + 5 - 6$

Step 1: Perform the calculation in the brackets first.
Step 2: Multiply or divide whichever comes first.
Step 3: Add or subtract whichever comes first.

$(1 + 2) \div 3 \times 4 + 5 - 6 = 3$

$1 + 2 = 3$
$3 \div 3 = 1$
$1 \times 4 = 4$
$4 + 5 = 9$
$9 - 6 = 3$

Year 6

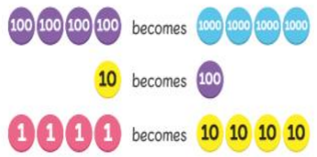
Multiplication

Multiplying by multiples of 10

Using counters

Using informal jottings

1 $414 \times 10 =$ ☐



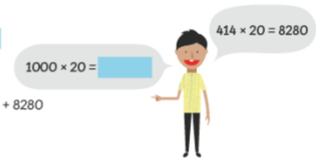
$414 \times 10 = 4000 + 100 + 40$
$= 4140$

2 $414 \times 20 =$ ☐

Method 1
$414 \times 10 = 4140$
$414 \times 20 = 4140 + 4140$
$= 8280$

Method 2
$414 \times 20 = 414 \times 2 \times 10$
$= 828 \times 10$
$= 8280$

3 $1414 \times 20 =$ ☐



Method 1
$1414 \times 20 = 20\,000 + 8280$
$= 28\,280$

Method 2
$1414 \times 10 = 14\,140$
$1414 \times 20 = 14\,140 \times 2$
$= 28\,280$

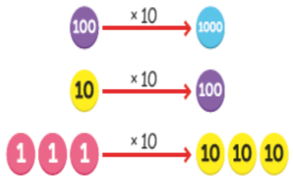
Method 3
$1414 \times 20 = 1414 \times 2 \times 10$
$= 2828 \times 10$
$= 28\,280$

Building to written methods

Using counters

using partitioning

1 $20 \times 113 =$ ☐

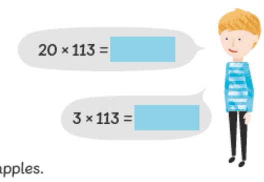


$10 \times 113 = 1130$
$20 \times 113 = 2260$

2 $23 \times 113 =$ ☐

$20 \times 113 = 2260$
$3 \times 113 = 339$
$23 \times 113 = 2599$

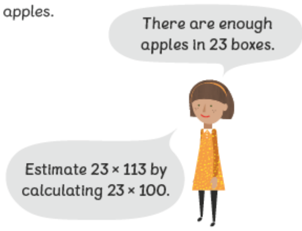
23 boxes contain 2599 apples.



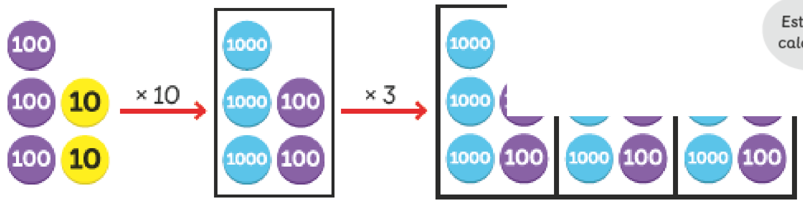
3 $23 \times 113 =$ ☐

$$\begin{array}{r} 113 \\ \times\ 20 \\ \hline 2260 \end{array} \quad \begin{array}{r} 113 \\ \times\ 3 \\ \hline 339 \end{array} \longrightarrow \begin{array}{r} 113 \\ \times\ 23 \\ \hline 2260 \\ +\ 339 \\ \hline 2599 \end{array} \quad \begin{array}{r} 113 \\ \times\ 23 \\ \hline 339 \\ +\ 2260 \\ \hline 2599 \end{array}$$

23 boxes contain 2599 apples.



1 $320 \times 31 =$ ☐



$320 \times 10 = 3200$ $3200 \times 3 = 9600$

$320 \times 30 = 9600$
$320 \times 1 = 320$
$320 \times 31 = 9920$

2 $320 \times 31 =$ ☐

$$\begin{array}{r} 320 \\ \times\ 30 \\ \hline 9600 \end{array} \quad \begin{array}{r} 320 \\ \times\ 1 \\ \hline 320 \end{array} \longrightarrow \begin{array}{r} 320 \\ \times\ 31 \\ \hline 9600 \\ +\ 320 \\ \hline 9920 \end{array}$$

$$\begin{array}{r} 320 \\ \times\ 30 \end{array} \quad \begin{array}{r} 320 \\ \times\ 1 \end{array} \longrightarrow \begin{array}{r} 320 \\ \times\ 31 \end{array}$$

Formal written method

5 $1320 \times 31 = $ ____

$$
\begin{array}{r}
1\;3\;2\;0 \\
\times \qquad 3\;1 \\
\hline
1\;3\;2\;0 \\
+\;3\;9\;6\;0\;0 \\
\hline

\end{array}
$$

$1\,3\,2\,0 \rightarrow 1320 \times 1$

$3\,9\,6\,0\,0 \rightarrow 1320 \times 30$

$1320 \times 1 = 1320$

$1320 \times 10 = 13\,200$

$1320 \times 30 = 39\,600$

Using known facts

1 $114 \times 12 = $ ____ $\qquad 114 \times 24 = $ ____

100 10 1 1 1 1

$114 \times 10 = 1140$
$114 \times \;\;2 = \;\;228$
$114 \times 12 = 1368$

$114 \times 20 = 2280$
$114 \times \;\;4 = \;\;456$
$114 \times 24 = 2736$

$$
\begin{array}{r}
{}^{1} \\
1\;1\;4 \\
\times \quad 4 \\
\hline
\square\;6 \\
\hline
\end{array}
$$

I can also use 114 × 12

2 $114 \times 24 = $ ____

100
10
1 1 1 1

$100 \times 24 = 2400$
$10 \times 24 = \;\;240$
$4 \times 24 = \;\;\;\;96$
$114 \times 24 = 2736$

$2 \times 24 = 48$
$4 \times 24 = 96$

3 $114 \times 24 = $ ____

$$
\begin{array}{r}
1\;1\;4 \\
\times \quad 2\;0 \\
\hline
2\;2\;8\;0 \\
\hline
\end{array}
\qquad
\begin{array}{r}
1\;1\;4 \\
\times \quad 4 \\
\hline
4\;5\;6 \\
\hline
\end{array}
\qquad \rightarrow \qquad
\begin{array}{r}
1\;1\;4 \\
\times \quad 2\;4 \\
\hline
2\;2\;8\;0 \\
+\;\;4\;5\;6 \\
\hline
2\;7\;3\;6 \\
\hline
\end{array}
$$

$2\,2\,8\,0 \rightarrow 114 \times 20$

$4\,5\,6 \rightarrow 114 \times 4$

1 1 4 $\qquad$ 1 1 4 $\qquad$ 1 1 4
× 2 0 $\qquad$ × 4 $\qquad$ × 2 4

4 Given that $114 \times 24 = 2736$, find the value of 2114×24.

$$\begin{array}{ccc} & 2114 & \\ \swarrow & & \searrow \\ 114 & & 2000 \end{array}$$

$114 \times 24 = 2736$

$$\begin{aligned} & 2000 \times 24 \\ &= 24 \times 2000 \\ &= 24 \times 2 \text{ thousands} \\ &= 48 \text{ thousands} \\ &= 48\,000 \end{aligned}$$

$$\begin{aligned} 2114 \times 24 &= 48\,000 + 2736 \\ &= 50\,736 \end{aligned}$$

Estimate by calculating 2000×24.

3 $24 \times 568 = \boxed{}$

$$\begin{array}{r} {\scriptstyle 2\ 3\ \ } \\ 5\ 6\ 8 \\ \times\ \ \ 2\ 4 \\ \hline 2\ 2\ 7\ 2 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ {\scriptstyle 2\ 3\ \ } \\ 5\ 6\ 8 \\ \times\ \ \ 2\ 4 \\ \hline 2\ 2\ 7\ 2 \\ 1\ 1\ 3\ 6\ 0 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ {\scriptstyle 2\ 3\ \ } \\ 5\ 6\ 8 \\ \times\ \ \ 2\ 4 \\ \hline 2\ 2\ 7\ 2 \\ +\ 1\ 1\ 3\ 6\ 0 \\ \hline 1\ 3\ 6\ 3\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 5\ 6\ 8 \\ \times\ \ \ \ 4 \\ \hline \end{array} \qquad \begin{array}{r} 5\ 6\ 8 \\ \times\ \ 2\ 0 \\ \hline \end{array}$$

Add the products.

$24 \times 568 = 13\,632$

Estimate $20 \times 600 = \boxed{}$

4 $24 \times 2568 = \boxed{}$

$$\begin{array}{r} {\scriptstyle 2\ 2\ 3\ \ } \\ 2\ 5\ 6\ 8 \\ \times\ \ \ \ 2\ 4 \\ \hline 1\ 0\ 2\ 7\ 2 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ {\scriptstyle 2\ 2\ 3\ \ } \\ 2\ 5\ 6\ 8 \\ \times\ \ \ \ 2\ 4 \\ \hline 1\ 0\ 2\ 7\ 2 \\ 5\ 1\ 3\ 6\ 0 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ {\scriptstyle 2\ 2\ 3\ \ } \\ 2\ 5\ 6\ 8 \\ \times\ \ \ \ 2\ 4 \\ \hline 1\ 0\ 2\ 7\ 2 \\ +\ 5\ 1\ 3\ 6\ 0 \\ \hline 6\ 1\ 6\ 3\ 2 \\ \hline \end{array}$$

2568×4

2568×20

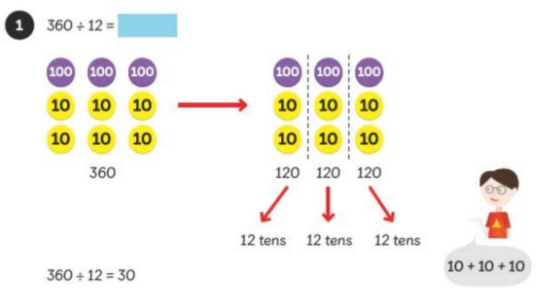
Add the products.

$24 \times 2568 = 61\,632$

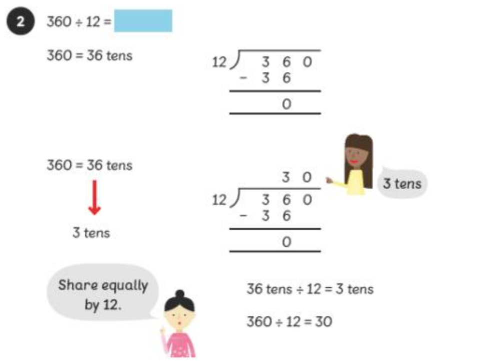
Estimate $20 \times 3000 = \boxed{}$

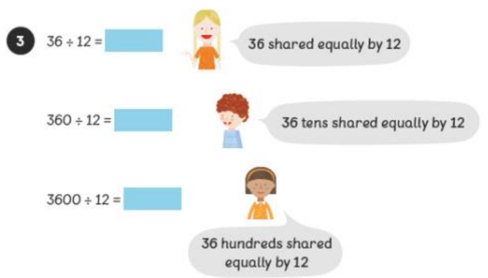
YEAR 6
Division

Using counters

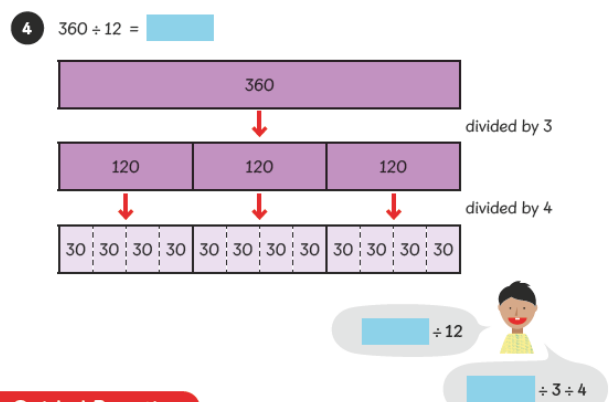


Using known facts





Using bar models



Using Partitioning

1. 3296 ÷ 32 = ☐

3296
3200 96

```
32 ) 3 2 9 6
   - 3 2 0 0
         9 6
       - 9 6
           0
```

It is possible to share £3296 equally among 32 people. Each would receive £103.

3296
3200 96
100 3

```
        1 0 3
32 ) 3 2 9 6
   - 3 2 0 0  → 3200 ÷ 32 = 100
         9 6
       - 9 6  → 96 ÷ 32 = 3
           0
```

2. 7192 ÷ 31 = ☐

7192
6200 992
930 62

→

7192
6200 992
200 teams
930 62
30 teams 2 teams

7192 ÷ 31 = 232

We can put 7192 participants into 232 teams of 31.

Using grouping

3. 7192 ÷ 31 = ☐

```
31 ) 7 1 9 2
   - 3 1 0 0   100
     4 0 9 2
   - 3 1 0 0   100
       9 9 2
     - 3 1 0    10
       6 8 2
     - 3 1 0    10
       3 7 2
     - 3 1 0    10
         6 2
       - 6 2     2
           0
```

```
31 ) 7 1 9 2
   - 6 2 0 0  → 6200 ÷ 31 = 200
       9 9 2
     - 9 3 0  → 930 ÷ 31 = 30
         6 2
       - 6 2  → 62 ÷ 31 = 2
           0
```


Formal written method

```
78 ) 8 5 8
   - 7 8 0
       7 8
     - 7 8
         0
```

Recording with remainders

As a number

Each tray contains 108 apricots.

$4 \times 108 = 432$

$500 - 432 = 68$

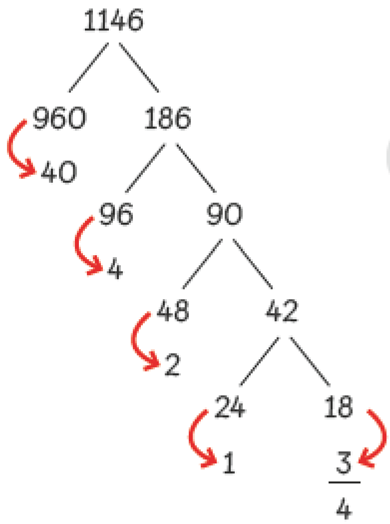
$500 \div 108 = 4$ remainder 68

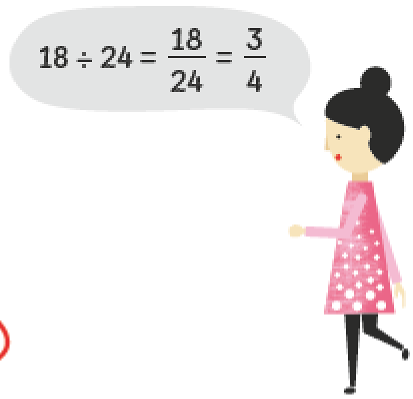
4 trays are needed to pack 500 apricots.

There are 68 apricots left over.

As fractions and decimals

1 £1146 ÷ 24 =





£1146 ÷ 24 = £47.75